ROHINI COLLEGE OF ENGINEERING AND TECHNOLOGY,

PALKULAM

Department of Electrical and Electronics Engineering

Training and Placement Certification Course

TRP 1501

APTITUDE SKILL – PHASE II

Course Instructor

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21. SIMPLE INTEREST

IMPORTANT FACTS AND FORMULAE

1.. **Principal**: The money borrowed or lent out for a certain period is called the **principal** or the **sum.**

2**. Interest**: Extra money paid for using other's money is called **interest.**

3. **Simple Interest (S.I.)** : If the interest on a sum borrowed for a certain period is reckoned uniformly, then it is called **simple interest.**

Let Principal = P, Rate = R% per annum (p.a.) and Time = T years. Then,

(i) S.I. = (P*R*T)/100

(ii) P=(100*S.I)/(R*T) ;R=(100*S.I)/(P*T) and T=(100*S.I)/(P*R)

SOLVED EXAMPLES

Ex. 1. **Find the simple interest on Rs. 68,000 at 16 2/3% per annum for 9 months.**

Sol. P = Rs.68000,R = 50/3% p.a and T = 9/12 years = 3/4years.

$\therefore$ S.I. = (P*R*T)/100 = Rs.(68,000*(50/3)*(3/4)*(1/100))= Rs.8500

Ex. 2. Find the simple interest on Rs. 3000 at 6 1/4% per annum for the period from 4th Feb., 2005 to 18th April, 2005.

Sol. Time = (24+31+18)days = 73 days = 73/365 years = 1/5 years.

P = Rs.3000 and R = 6 ¼ %p.a = 25/4%p.a

$\therefore$S.I. = Rs.(3,000*(25/4)*(1/5)*(1/100))= Rs.37.50.

Remark : The day on which money is deposited is not counted while the day on which money is withdrawn is counted .

Ex. 3. A sum at simple interests at 13 ½ % per annum amounts to Rs.2502.50 after 4 years find the sum.

Sol. Let sum be Rs. x then , S.I.=Rs.(x*(27/2) *4*(1/100)) = Rs.27x/50

$\therefore$ amount = $\left(\text{Rs. } x+(27x/50)\right)$ = Rs.77x/50

$\therefore$ 77x/50 = 2502.50 $\Leftrightarrow$ x = $\frac{2502.50 * 50}{77}$ = 1625

Hence , sum = Rs.1625.

Ex. 4. A sum of Rs. 800 amounts to Rs. 920 in 8 years at simple intere interest rate is increased by 8%, it would amount to bow mucb ?

Sol. S.l. = Rs. (920 - 800) = Rs. 120; p = Rs. 800, T = 3 yrs. _

. R = $\left((100 \text{ x } 120)/(800*3)\right)$ % = 5%.

New rate = (5 + 3)% = 8%.
New S.l. = Rs. (800*8*3)/100 = Rs. 192.

: New amount = Rs.(800+192) = Rs. 992.

Ex. 5. Adam borrowed some money at the rate of 6% p.a. for the first two years , at the rate of 9% p.a. for the next three years , and at the rate of 14% p.a. for the period beyond five years. 1£ he pays a total interest of Rs. 11, 400 at the end of nine years how much money did he borrow ?

Sol. Let the sum borrowed be x. Then,

(x*2*6)/100 + (x*9*3)/100 + (x*14*4)/100 = 11400

$\Leftrightarrow$ $\left(3x/25 + 27x/100 + 14x / 25\right)$ = 11400 $\Leftrightarrow$ 95x/100 = 11400 $\Leftrightarrow$ x = (11400*100)/95 = 12000.

Hence , sum borrowed = Rs.12,000.

Ex. 6. A certain sum of money amounts to Rs. 1008 in 2 years and to Rs.1164 in 3 ½ years. Find the sum and rate of interests.

Sol.. S.I. for 1 ½ years = Rs.(1164-1008) = Rs.156.

S.l. for 2 years = Rs.(156*(2/3)*2)=Rs.208

Principal = Rs. (1008 - 208) = Rs. 800.

Now, P = 800, T = 2 and S.l. = 208.

Rate =(100* 208)/(800*2)% = 13%

Ex. 7. At what rate percent per annum will a sum of money double in 16 years.

Sol.. Let principal = P. Then, S.I. = P and T = 16 yrs.

$\therefore$ Rate = (100 x P)/(P*16)% = 6 ¼ % p.a.

Ex. 8. The simple interest on a sum of money is 4/9 of the principal .Find the rate percent and time, if both are numerically equal.

Sol. Let sum = Rs. x. Then, S.I. = Rs. 4x/9

Let rate = R% and time = R years.

Then, (x*R*R)/100=4x/9 or R^2 =400/9 or R = 20/3 = 6 2/3.

$\therefore$ Rate = 6 2/3 % and Time = 6 2/3 years = 6 years 8 months.

Ex. 9. The simple interest on a certain sum of money for 2 1/2 years at 12% per annum is Rs. 40 less tban the simple interest on the same sum for 3 ½ years at 10% per annum. Find the sum.

Sol. Let the sum be Rs. x Then, ((x*10*7)/(100*2)) − ((x*12*5)/(100*2)) = 40

$\Leftrightarrow$ (7x/20)-(3x/10)=40
$\Leftrightarrow$ x = (40 * 20) = 800.

Hence, the sum is Rs. 800.

Ex. 10. A sum was put at simple interest at a certain rate for 3 years. Had it been put at 2% higher rate, it would have fetched Rs. 360 more. Find the sum.

Sol. Let sum = P and original rate = R.

Then, [(P*(R+2)*3)/100] − [(P*R*3)/100] = 360.

$\Leftrightarrow$ 3PR + 6P - 3PR = 36000 $\Leftrightarrow$ 6P=36000 $\Leftrightarrow$ P=6000

Hence, sum = Rs. 6000.

Ex. 11. What annual instalment will discharge a debt of Rs. 1092 due in 3 years at 12% simple interest?

Sol .Let each Instalment be Rs. x

Then, (x+ ((x*12*1)/100)) + (x+ ((x*12*2)/100)) + x = 1092

$\Leftrightarrow ((28x/25) + (31x/25) + x) = 1092 \quad \Leftrightarrow (28x+31x+25x)=(1092*25)$

$\Leftrightarrow$ x= (1092*25)/84 = Rs.325.

$\therefore$ Each instalment = Rs. 325.

Ex. 12. A sum of Rs. 1550 is lent out into two parts, one at 8% and another one at 6%. If the total annual income is Rs. 106, find the money lent at each rate.

Sol. Let the sum lent at 8% be Rs. x and that at 6% be Rs. (1550 - *x).*

$\therefore \big((x*8*1)/100\big) + \big((1550-x)*6*1\big)/100=106$

$\Leftrightarrow 8x + 9300 - 6x=10600 \Leftrightarrow 2x = 1300 \Leftrightarrow x = 650.$

$\therefore$ Money lent at 8% = Rs. 650. Money lent at 6% = Rs. (1550 - 650) = Rs. 900.

22.COMPOUND INTEREST

Compound Interest: Sometimes it so happens that the borrower and the lender agree to fix up a certain unit of time, say *yearly* or *half-yearly* or *quarterly* to settle the previous account.

In such cases, the amount after first unit of time becomes the principal for the second unit,the amount after second unit becomes the principal for the third unit and so on.

After a specified period, *the difference between the amount and the money borrowed is called the* Compound *Interest (abbreviated* as *C.I.) for that period.*

IMPORTANT FACTS AND FORMULAE

Let Principal = P, Rate = R% per annum, Time = n years.
I. When interest is compound Annually:

Amount = $P(1+R/100)^n$

II. When interest is compounded Half-yearly:

Amount = $P[1+(R/2)/100]^{2n}$

III. When interest is compounded Quarterly:

Amount = $P[\ 1+(R/4)/100]^{4n}$

IV. When interest is compounded AnnuaI1y but time is in fraction, say 3(2/5) years.

Amount = $P(1+R/100)^3 \times (1+(2R/5)/100)$

V. When Rates are different for different years, say Rl%, R2%, R3% for 1st, 2nd and 3rd year respectively.

Then, Amount = $P(1+R_1/100)(1+R_2/100)(1+R_3/100)$

VI. Present worth of Rs.x due *n* years hence is given by :

Present Worth = $x/(1+(R/100))^n$

SOLVED EXAMPLES

Ex.1. ***Find compound interest* on *Rs. 7500* at 4*% per* annum *for* 2 *years, compounded* annually.**

Sol.

Amount = Rs [7500*(1+(4/100)^2] = Rs (7500 * (26/25) * (26/25)) = Rs. 8112.

therefore, C.I. = Rs. (8112 - 7500) = Rs. 612.

Ex. 2. ***Find compound interest* on *Rs. 8000* at 15*% per* annum *for* 2 *years* 4 *months, compounded annually*.**

Sol. Time = 2 years 4 months = 2(4/12) years = 2(1/3) years.

Amount = Rs'. [8000 X (1+(15/100))^2 X (1+((1/3)*15)/100)]

=Rs. [8000 * (23/20) * (23/20) * (21/20)]

= Rs. 11109.

:. C.I. = Rs. (11109 - 8000) = Rs. 3109.

Ex. 3. ***Find the compound interest* on *Rs. 10,000 in* 2 *years* at 4*% per* annum, *the interest being compounded half-yearly.*** **(S.S.C. 2000)**

Sol.

Principal = Rs. 10000; Rate = 2% per half-year; Time = 2 years = 4 half-years.

Amount =
Rs [10000 * (1+(2/100))^4] = Rs(10000 * (51/50) * (51/50) * (51/50) * (51/50))

= Rs. 10824.32.

:. C.I. = Rs. (10824.32 - 10000) = Rs. 824.32.

Ex. 4. ***Find the compound interest* on *Rs. 16,000* at *20% per* annum *for* 9 *months, compounded quarterly.***

Sol. Principal = Rs. 16000; Time = 9 months =3 quarters;
Rate = 20% per annum = 5% per quarter.

Amount = Rs. [16000 x (1+(5/100))^3] = Rs. 18522.

CJ. = Rs. (18522 - 16000) = Rs. 2522.

Ex. 5. ***If the simple interest*** **on a** ***sum*** **of** ***money*** **at 5%** ***per*** **annum** ***for*** **3** ***years is Rs. 1200, find the compound interest*** **on** ***the same sum for the*** **same** ***period at the same r*****ate.**

Sol.

Clearly, Rate = 5% p.a., Time = 3 years, S.I.= Rs. 1200. . .

So principal=RS [100*1200]/3*5=RS 8000

Amount = Rs. 8000 x [1 +5/100]^3 - = Rs. 9261.

.. C.I. = Rs. (9261 - 8000) = Rs. 1261.

Ex. 6. ***In what time will Rs. 1000 become Rs.*** **1331 at** ***10% per*** **annum** ***compounded annually?*** **(S.S.C. 2004)**

Sol.

Principal = Rs. 1000; Amount = Rs. 1331; Rate = 10% p.a. Let the time be n years. Then,
[1000 $(1+ (10/100))^n$] = 1331 or $(11/10)^n =$ (1331/1000) = $(11/10)^3$
n = 3 years.

Ex. 7. ***If Rs. 600 amounts*** **to** ***Rs. 683.20 in*** **two years** ***compounded*** **annually, find the rate of** ***interest per*** **annum.**

Sol. Principal = Rs. 500; Amount = Rs. 583.20; Time = 2 years.
Let the rate be R% per annum.. 'Then,

[500 $(1+(R/100)^2$] = 583.20 or [1+ $(R/100)]^2 =$ 5832/5000 = 11664/10000

[1+ $(R/100)]^2 = (108/100)^2$ or 1 + (R/100) = 108/100 or R = 8

So, rate = 8% p.a.

Ex. 8. ***If the compound interest*** **on a** ***certain sum*** **at 16 (2/3)%** ***to*** **3** ***years is Rs.1270, find the simple interest*** **on** ***the same sum*** **at** ***the same*** **rate and** ***f or the*** **same period.**

Sol. Let the sum be Rs. x. Then,

C.I. = [x * $(1 + ((50/(3*100))^3$ - x] = ((343x / 216) - x) = 127x / 216

127x /216 = 1270 or x = (1270 * 216) / 127 = 2160.

Thus, the sum is Rs. 2160

S.I. = Rs (2160 * (50/3) * 3 * (1 /100)) = Rs. 1080.

Ex. 9. ***The difference*** **between** ***the compound interest*** **and** ***simple interest*** **on a** ***certain sum*** **at 10%** ***per*** **annum** ***for*** **2** ***years is Rs.*** **631.** ***Find the sum.***

Sol. Let the sum be Rs. x. Then,

C.I. = $x(1 + (10/100))^2 - x = 21x / 100$,

S.I. = $((x * 10 * 2) / 100) = x / 5$

$(C.I) - (S.I) = ((21x / 100) - (x / 5)) = x / 100$

$(x / 100) = 632$ □ $x = 63100$.

Hence, the sum is Rs.63,100.

Ex. 10. ***The difference* between *the compound interest* and *the simple* interest *accrued* on an amount *of Rs. 18,000 in* 2 *years was Rs. 405. What* was *the* rate *of interest p.c.p.a.* ?** **(Bank P.O. 2003)**

Sol. Let the *rate* be R% p.a. then,

$[18000(1 + (R/100)^2) - 18000] - ((18000 * R * 2)/100) = 405$

$18000[(100 + (R/100)^2 / 10000) - 1 - (2R/100)] = 405$

$18000[((100 + R)^2 - 10000 - 200R) / 10000] = 405$

$9R^2 / 5 = 405$ □ $R^2 = ((405 * 5) / 9) = 225$

$R = 15$.

Rate = 15%.

Ex. 11. ***Divide Rs. 1301 between A and B, so that the amount* of *A after* 7 *years is equal to the amount* of *B after* 9 *years, the interest being compounded at* 4*% per annum.***

Sol. Let the two parts be Rs. x and Rs. (1301 - *x).*

$x(1+4/100)^7 = (1301-x)(1+4/100)^9$

$x/(1301-x) = (1+4/100)^2 = (26/25*26/25)$

$625x = 676(1301-x)$

$1301x = 676*1301$

$x = 676$.

So,the parts are rs.676 and rs.(1301-676)i.e rs.676 and rs.625.

Ex.12. a certain sum amounts to rs.7350 in 2 years and to rs.8575 in 3 years.find the sum and rate percent.

S.I on rs.7350 for 1 year=rs.(8575-7350)=rs.1225.

*Rate=(100*1225/7350*1)%=16 2/3%*

Let the sum be rs.x.then,

$X(1+50/3*100)^2 = 7350$

$X*7/6*7/6 = 7350$

$X = (7350*36/49) = 5400$.

Sum=rs.5400.

Ex.13.a sum of money amounts to rs.6690 after 3 years and to rs.10,035 after 6 years on compound interest.find the sum.

Sol. *Let the sum be rs.P.then*

$P(1+R/100)^3=6690...(i)$ *and* $P(1+R/100)^6=10035...(ii)$

On dividing,we get $(1+R/100)^3=10025/6690=3/2.$

Substituting this value in (i),we get:

$P*3/2=6690$ *or* $P=(6690*2/3)=4460$

Hence,the sum is rs.4460.

Ex.14. a sum of money doubles itself at compound interest in 15 years.in how many years will it beco,e eight times?

$P(1+R/100)^{15}=2P$

$(1+R/100)^{15}=2P/P=2$

LET $P(1+R/100)^n=8P$

$(1+R/100)^n=8=2^3=\{(1+R/100)^{15}\}^3$ [USING (I)]

$(1+R/100)^N=(1+R/100)^{45}$

$n=45.$

Thus,the required time=45 years.

Ex.15.What annual payment will discharge a debt of Rs.7620 due in 3years at 16 2/3% per annum interest?

Sol. Let each installment beRs.x.

Then,(P.W. of Rs.x due 1 year hence)+(P>W of Rs.x due 2 years hence)+(P.W of Rs. X due 3 years hence)=7620.

$\therefore$ $x/(1+(50/3*100))+ x/(1+(50/3*100))^2 + x/(1+(50/3*100))^3=7620$

$\Leftrightarrow(6x/7)+(936x/49)+(216x/343)=7620.$

$\Leftrightarrow 294x+252x+216x=7620*343.$

$\Leftrightarrow x=(7620*343/762)=3430.$

$\therefore$ Amount of each installment=Rs.3430.

23. LOGARITHMS

IMPORTANT FACTS AND FORMULAE

I. **Logarithm**: If a is a positive real number, other than 1 and $a^m = X$, then we write: **m =** $\boldsymbol{log_a}$ **x** and we say that the value of log x to the base *a* is m.

Example:

(i) $10^3 = 1000 \Rightarrow \log_{10} 1000 = 3$

(ii) $2^{-3} = 1/8 \Rightarrow \log_2 1/8 = -3$

(iii) $3^4 = 81 \Rightarrow \log_3 81 = 4$

(iiii) $(.1)^2 = .01 \Rightarrow \log_{(.1)} .01 = 2$.

II. Properties of Logarithms:

1. $\log_a(xy) = \log_a x + \log_a y$

2. $\log_a (x/y) = \log_a x - \log_a y$

3. $\log_x x = 1$

4. $\log_a 1 = 0$

5. $\log_a(x^p) = p(\log_a x)$ 1

6. $\log_a x = 1/\log_x a$

7. $\log_a x = \log_b x/\log_b a = \log\_x/\log a$.

Remember: When base is not mentioned, it is taken as 10.

II. **Common Logarithms:**

Logarithms to the base 10 are known as common logarithms.

III. The logarithm of a number contains two parts, namely *characteristic* and *mantissa.*

Characteristic: The integral part of the logarithm of a number is called its ***characteristic.***

Case I: When the number is greater than 1.

In this case, the characteristic is one less than the number of digits in the left of the decimal point in the given number.

Case II: When the number is less than 1.

In this case, the characteristic is one more than the number of zeros between the decimal point and the first significant digit of the number and it is negative.

Instead of - 1, - 2, etc. we write, $\bar{1}$ (one bar), $\bar{2}$ (two bar), etc.

Example:

Number	*Characteristic*	*Number*	*Characteristic*
348.25	2	0.6173	$\bar{1}$
46.583	1	0.03125	$\bar{2}$
9.2193	0	0.00125	$\bar{3}$

Mantissa: The decimal part of the logarithm of a number is known is its ***mantissa.*** For mantissa, we look through log table.

SOLVED EXAMPLES

1.Evaluate:

(1)**log**$_3$ 27

(2)**log**$_7$ (1/343)

(3)**log**$_{100(0.01)}$

SOLUTION:

(1) let $\log_3 27 = 3^3$ or n=3.

ie, $\log_3 27 = 3$.

(2) Let $\log_7 (1\backslash 343) = n$.

Then ,$7^n = 1/343$

$= 1/7^3$

n = -3.

ie,

$\log_7(1\backslash 343) = -3$.

(3) let $\log_{100}(0.01) = n$.

Then,. (100) = 0.01 = 1 /100=100 $^{-1}$ 0r n=-1

EX.2. evaluate

(i) $\log_7 1=0$ (ii)$\log_{34} 34$ (iii)$36^{\log_6 4}$

solution:

i) we know that $\log_a 1=0$,so $\log_7 1=0$.

ii) we know that $\log_a a=1$,so $\log_{34} 34=0$.

iii) We know that $a^{\log_6 x} = x$.

now $36^{\log_6 4} = (6^2)^{\log_6 4} = 6^{\log_6 (16)=16.}$

Ex.3.if $\log_{\sqrt{8}} x=3\ (1/3)$, find the value of x.

$\log_{\sqrt{8}} x=10/3$,$x=(\sqrt{8})^{10/3}=(2^{3/2})^{10/3}=2^{(3/2*10/3)}=2^5=32$.

Ex.4:Evaluate: (i) $\log_5 3*\log_{27} 25$ (ii) $\log 27 - \log_{27} 9$

(i)$\log_5{}^3 * \log_{27} 25=(\log 3/\log 5)*(\log 25/\log 27)$
$=(\log 3/\log 5)*(\log 5^2*\log 3^3)$
$=(\log 3/\log 5)*(2\log 5/3\log 3)$
$=2/3$

(ii)Let $\log_9 27=n$
Then,
$9^n=27 \Leftrightarrow 3^{2n}=3^3 \Leftrightarrow 2n=3 \Leftrightarrow n=3/2$
Again, let $\log_{27} 9=m$
Then,
$27^m=9 \Leftrightarrow 3^{3m}=3^2 \Leftrightarrow 3m=2 \Leftrightarrow m=2/3$

$\Rightarrow \log_9 27- \log_{27} 9=(n-m)=(3/2-2/3)=5/6$

Ex 5. Simplify :(log 75/16-2 log 5/9+log 32/243)
Sol: log 75/16-2 log 5/9+log 32/243
= $\log 75/16-\log(5/9)^2+\log 32/243$
= log 75/16-log25/81+log 32/243
= log(75/16*32/243*81/25)=log 2

Ex. 6.Find the value of x which satisfies the relation
$\text{Log}_{10} 3+\log_{10}(4x+1)=\log_{10}(x+1)+1$
Sol: $\log_{10} 3+\log_{10}(4x+1)=\log_{10}(x+1)+1$
$\text{Log}_{10} 3+\log_{10}(4x+1)=\log_{10}(x+1)+\log_{10}(x+1)+\log_{10} 10$
$\text{Log}_{10}(3(4x+1))=\log_{10}(10(x+1))$
=3(4x+1)=10(x+1)=12x+3
=10x+10
=2x=7=x=7/2

Ex. 7.Simplify:[$1/\log_{xy(}xyz)+1/\log_{yz}(xyz)+1/\log_{zx}(xyz)$]
Given expression: $\log_{xyz} xy+ \log_{xyz} yz+ \log_{xyz} zx$
$=\log_{xyz}(xy*yz*zx)=\log_{xyz}(xyz)^2$
$2\log_{xyz}(xyz)=2*1=2$

Ex.8.If $\log_{10} 2=0.30103$,find the value of $\log_{10} 50$.
Soln. $\log_{10} 50=\log_{10}(100/2)=\log_{10} 100-\log_{10} 2=2-0.30103=1.69897$.

Ex 9.If log 2=0.3010 and log 3=0.4771,find the values of:

i) log 25 ii)log 4.5

Soln.

i) log 25=log(100/4)=log 100-log 4=2-2log 2=(2-2*.3010)=1.398.

ii) log 4.5=log(9/2)=log 9-log 2=2log 3-log 2
=(2*0.4771-.3010)=.6532

Ex.10. If log 2=.30103,find the number of digits in 2^{56}.

Soln. log 2^{56} =56log2=(56*0.30103)=16.85768.

Its characteristics is 16.

Hence,the number of digits in 2^{56} is 17

24 .AREA

FUNDEMENTAL CONCEPTS

I.RESULTS ON TRIANGLES:
1.Sum of the angles of a triangle is 180 degrees.

2.Sum of any two sides of a triangle is greater than the third side.

3.Pythagoras theorem:
In a right angle triangle,
(Hypotenuse)^2 = (base)^2 + (Height)^2

4.The line joining the midpoint of a side of a triangle to the opposite vertex is called the

MEDIAN
5.The point where the three medians of a triangle meet is called **CENTROID**.
Centroid divides each of the medians in the ratio 2:1.

6.In an isosceles triangle, the altitude from the vertex bi-sects the base

7.The median of a triangle divides it into two triangles of the same area.

8.Area of a triangle formed by joining the midpoints of the sides of a given triangle is one-fourth of the area of the given triangle.

II.RESULTS ON QUADRILATERALS:
1. The diagonals of a parallelogram bisects each other .
2. Each diagonal of a parallelogram divides it into two triangles of the same area
3. The diagonals of a rectangle are equal and bisect each other.
4. The diagonals of a square are equal and bisect each other at right angles.
5. The diagonals of a rhombus are unequal and bisect each other at right angles.
6. A parallelogram and a rectangle on the same base and between the same parallels are equal in area.
7. Of all the parallelograms of a given sides , the parallelogram which is a rectangle has the greatest area.

IMPORTANT FORMULAE

I.1.Area of a rectangle=(length*breadth)
Therefore length = (area/breadth) and breadth=(area/length)
2.Perimeter of a rectangle = 2*(length+breadth)
II.Area of a square = (side)^2 =1/2(diagonal)^2
III Area of four walls of a room = 2*(length + breadth)*(height)
IV 1.Area of the triangle=1/2(base*height)
2. Area of a triangle = (s*(s-a)(s-b)(s-c))^(1/2), where a,b,c are the sides of a triangle

and s= ½(a+b+c)

3.Area of the equilateral triangle =((3^1/2)/4)*(side)^2

4.Radius of incircle of an equilateral triangle of side a=a/2(3^1/2)

5.Radius of circumcircle of an equilateral triangle of side a=a/(3^1/2)

6.Radius of incircle of a triangle of area del and semiperimeter S=del/S

V.1.Area of the parellogram =(base *height)

2.Area of the rhombus=1/2(product of the diagonals)

3.Area of the trapezium=1/2(size of parallel sides)*distance between them

VI 1.Area of a circle =pi*r^2,where r is the radius

2. Circumference of a circle = 2∏R.

3. Length of an arc = 2∏Rθ/(360) where θ is the central angle

4. Area of a sector = (1/2) (arc x R) = pi*R^2*θ/360.

VII. 1. Area of a semi-circle = (pi)*R^2.

2. Circumference of a semi-circle = (pi)*R.

SOLVED EXAMPLES

Ex.1. One side of a rectangular field is 15 m and one of its diagonals is 17 m. Find the area of the field.

Sol. Other side = $((17)^2 - (15)^{2)(1/2)}) = (289- 225)^{(1/2)} = (64)^{(1/2)} = 8$ m.

Area = (15 x 8) m^2 = 120 m^2.

Ex. 2. A lawn is in the form of a rectangle having its sides in the ratio 2: 3. The area of the lawn is (1/6) hectares. Find the length and breadth of the lawn.

Sol. Let length = 2x metres and breadth = 3x metre.

Now, area = (1/6)x 1000 m^2 = 5000/3m^2

So, 2x * *3x* = 5000/3 <=> x^2 = 2500/9 <=> *x* = 50/3

therefore Length = 2x = (100/3) m = 33(1/3) m and Breadth = *3x* = 3(50/3) m = 50m.

Ex. 3. Find the cost of carpeting a room 13 m long and 9 m broad with a carpet 75 cm wide at the rate of Rs. 12.40 per square metre.

Sol. Area of the carpet = Area of the room = (13 * 9) m^2 = 117 m^2.

Length of the carpet = (area/width) = 117 *(4/3) m = 156 m.

Therefore Cost of carpeting = Rs. (156 * 12.40) = Rs. 1934.40.

Ex. 4. If the diagonal of a rectangle is 17 cm long and its perimeter is 46 cm, find the area of the rectangle. .

Sol. Let length = x and breadth = *y*. Then,

$2(x + y) = 46$ or $x + y = 23$ and $x^2 + y^2 = (17)^2 = 289$.

Now, $(x + y)^2 = (23)^2 \Leftrightarrow (x^2 + y^2) + 2xy = 529 \Leftrightarrow 289 + 2xy = 529 \Leftrightarrow xy=120$

Area = xy = 120 cm^2.

Ex. 5. The length of a rectangle is twice its breadth. If its length is decreased by 5 cm and breadth is increased by 5 cm, the area of the rectangle is increased by 75 sq. cm. Find the length of the rectangle.

Sol. Let breadth = x. Then, length = *2x.* Then,
$(2x - 5)(x + 5) - 2x * x = 75 \Leftrightarrow 5x - 25 = 75 \Leftrightarrow x = 20$.
:. Length of the rectangle = 20 cm.

Ex. 6. In measuring the sides of a rectangle, one side is taken 5% in excess, and the other 4% in deficit. Find the error percent in the area calculated from these measurements. (M.B.A. 2003)

Sol. Let x and *y* be the sides of the rectangle. Then, Correct area = xy.
Calculated area = (105/100)*x * (96/100)*y = (504/500)(xy)
Error In measurement = (504/500)xy- *xy* = (4/500)xy
Error % = [(4/500)*xy* *(1/xy) *100] % = (4/5) % = 0.8%.

Ex. 7. A rectangular grassy plot 110 m. by 65 m has a gravel path 2.5 m wide all round it on the inside. Find the cost of gravelling the path at 80 paise per sq. metre.

Sol. Area of the plot = (110 x 65) m^2 = 7150 m^2
Area of the plot excluding the path = [(110 - 5) * (65 - 5)] m^2 = 6300 m^2.
Area of the path = (7150 - 6300) m^2 = 850 m^2.

Cost of gravelling the path = Rs.850 * (80/100)= Rs. 680

Ex. 8. The perimeters of two squares are 40 cm and 32 cm. Find the perimeter of a third square whose area is equal to the difference of the areas of the two squares. (S.S.C. 2003)

Sol. Side of first square = (40/4) = 10 cm;
Side of second square = (32/4)cm = 8 cm.
Area of third square = $[(10)^2 - (8)^2]$ cm^2 = (100 - 64) cm^2 = 36 cm^2.
Side of third square = $(36)^{(1/2)}$ cm = 6 cm.
Required perimeter = (6 x 4) cm = 24 cm.

Ex. 9. A room 5m 55cm long and 3m 74 cm broad is to be paved with square tiles. Find the least number of square tiles required to cover the floor.

Sol. Area of the room = (544 x 374) cm^2.
Size of largest square tile = H.C.F. of 544 cm and 374 cm = 34 cm.
Area of 1 tile = (34 x 34) cm^2.
Number of tiles required =(544*374)/(34*34)=176

Ex. 10. Find the area of a square, one of whose diagonals is 3.8 m long.
Sol. Area of the square = $(1/2)*(\text{diagonal})^2 = [(1/2)*3.8*3.8]m^2 = 7.22\ m^2$.

Ex. 11. The diagonals of two squares are in the ratio of 2 : 5. Find the ratio of their areas. (Section Officers', 2003)
Sol. Let the diagonals of the squares be 2x and 5x respectively.

Ratio of their areas = $(1/2)*(2x)^2 : (1/2)*(5x)^2 = 4x^2 : 25x^2 = 4 : 25$.

Ex.12. If each side of a square is increased by 25%, find the percentage change in its area.

Sol. Let each side of the square be a. Then, area = a^2.

New side $=(125a/100) =(5a/4)$. New area $= (5a/4)^2 =(25a^2)/16$.
Increase in area $= ((25a^2)/16)-a^2 =(9a^2)/16$.
Increase% $= [((9a^2)/16)*(1/a^2)*100]\ \% = 56.25\%$.

Ex. 13. If the length of a certain rectangle is decreased by 4 cm and the width is increased by 3 cm, a square with the same area as the original rectangle would result. Find the perimeter of the original rectangle.

Sol. Let x and y be the length and breadth of the rectangle respectively.
Then, $x - 4 = y + 3$ or $x - y = 7$ ----(i)
Area of the rectangle $=xy$; Area of the square $= (x - 4)(y + 3)$
$(x - 4)(y + 3) = xy \iff 3x - 4y = 12$ ----(ii)
Solving *(i)* and *(ii),* we get $x = 16$ and $y = 9$.
Perimeter of the rectangle $= 2(x + y) = [2(16 + 9)]$ cm = 50 cm.

Ex. 14. A room is half as long again as it is broad. The cost of carpeting the at Rs. 5 per sq. m is Rs. 270 and the cost of papering the four walls at Rs. 10 per m^2 is Rs. 1720. If a door and 2 windows occupy 8 sq. m, find the dimensions of the room.

Sol. Let breadth = x metres, length = <u>*3x*</u> metres, height = H metres.

Area of the floor=(Total cost of carpeting)/(Rate/m^2)=$(270/5)m^2=54m^2$.
$x*(3x/2) = 54 \iff x^2 = (54*2/3) = 36 \iff x = 6$.
So, breadth = 6 m and length $=(3/2)*6 = 9$ m.
Now, papered area $= (1720/10)m^2 = 172\ m^2$.
Area of 1 door and 2 windows = $8\ m^2$.
Total area of 4 walls $= (172 + 8)\ m^2 = 180\ m^2$

$2*(9+6)*H = 180 \Leftrightarrow H = 180/30 = 6$ m.

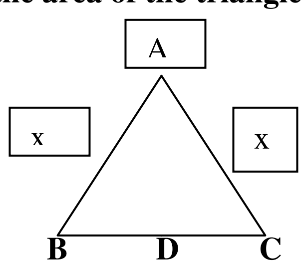
Ex. 15. Find the area of a triangle whose sides measure 13 cm, 14 cm and 15 cm.

Sol. Let a = 13, b = 14 and c = 15. Then, S = (1/2)$(a + b$ + c) = 21.
(s- a) = 8, (s - b) = 7 and (s - c) = 6.
Area = $(s(s-a)(s-b)(s-c))^{(1/2)} = (21 * 8 * 7 * 6)^{(1/2)} = 84\ cm^2$.

Ex. 16. Find the area of a right-angled triangle whose base is 12 cm and hypotenuse is 13cm.
Sol. Height of the triangle = $[(13)^2 - (12)^2]^{(1/2)}$ cm = $(25)^{(1/2)}$ cm = 5 cm.
Its area = (1/2)* Base * Height = ((1/2)*12 * 5) cm^2 = 30 cm^2.

Ex. 17. The base of a triangular field is three times its altitude. If the cost of cultivating the field at Rs. 24.68 per hectare be Rs. 333.18, find its base and height.
Sol. Area of the field = Total cost/rate = (333.18/25.6)hectares = 13.5 hectares
⇔ (13.5 x 10000) m^2 = 135000 m^2.
Let altitude = x metres and base = 3x metres.
Then, $(1/2)* 3x * x = 135000 \Leftrightarrow x^2 = 90000 \Leftrightarrow x = 300$.
Base = 900 m and Altitude = 300 m.

Ex. 18. The altitude drawn to the base of an isosceles triangle is 8 cm and the perimeter is 32 cm. Find the area of the triangle.



Sol. Let ABC be the isosceles triangle and AD be the altitude.
Let AB = AC = x. Then, BC = (32 - $2x$).
Since, in an isosceles triangle, the altitude bisects the base,
so BD = DC = (16 - x).
In *triangle* ADC, $AC^2 = AD + DC^2 \Rightarrow x^2 = (8^2) + (16-x)^2$
$\Rightarrow 32x = 320 \Rightarrow x = 10$.
BC = (32- $2x$) = (32 - 20) cm = 12 cm.
Hence, required area = ((1/2)x*BC * AD) = ((1/2)*12 *10)cm^2 = 60 cm^2.

Ex. 19. Find the length of the altitude of an equilateral triangle of side 3√3 cm.

Sol. Area of the triangle = <u>(√3/4)</u> x *(3 √3)2* = <u>*27 √3.*</u> Let the height be *h.*

Then, (1/2) x 3√3 x *h* = (27√3/4) X(2/√3) = 4.5 cm.

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Ex. 20. In two triangles, the ratio of the areas is 4 : 3 and the ratio of their heights is 3 : 4. Find the ratio of their bases.

Sol. Let the bases of the two triangles be x and y and their heights be *3h* and *4h* respectively. Then,

((1/2) X x X 3h)/(1/2) X y X 4h) =4/3 ⇔ x/y =(4/3 X 4/3)=16/9

Required ratio = 16 : 9.

Ex.21. The base of a parallelogram is twice its height. If the area of the parallelogram is 72 sq. cm, find its height.

Sol. Let the height of the parallelogram be x. cm. Then, base = (2x*)* cm.

2x X x =72 ⇔ 2x^2 = 72 ⇔ X ^2=36 ⇔ x=6

Hence, height of the parallelogram = 6 cm.

Ex. 22. Find the area of a rhombus one side of which measures 20 cm and 01 diagonal 24 cm.

Sol. Let other diagonal = 2x cm.

Since diagonals of a rhombus bisect each_other at right angles, we have:

(20)2 = (12)2 + (x)2 _ *x* = √(20)2 – (12)2= √256= 16 cm. _I

So, other diagonal = 32 cm.

Area of rhombus = (1/2) x (Product of diagonals) = *((1/2)x* 24 x 32) cm^2 = 384 cm^2

Ex. 23. The difference between two parallel sides of a trapezium is 4 cm. perpendicular distance between them is 19 cm. If the area of the trapezium is 475 find the lengths of the parallel sides. (R.R.B. 2002)

Sol. Let the two parallel sides of the trapezium be *a* em and b em.

Then, *a* - *b* = 4

And, (1/2) x *(a + b)* x 19 = 475 ⇔ *(a + b)* =((475 x 2)/19) ⇔ *a* + *b* = 50

Solving (i) and *(*ii*),* we get: *a* = 27, *b* = 23.

So, the two parallel sides are 27 cm and 23 cm.

Ex. 24. Find the length of a rope by which a cow must be tethered in order tbat it may be able to graze an area of 9856 sq. metres. (M.A.T. 2003)

Sol. Clearly, the cow will graze a circular field of area 9856 sq. metres and radius equal to the length of the rope.

Let the length of the rope be R metres.

Then, ∏(R)^2 = (9856 X (7/22)) = 3136⇔ R = 56.

Length of the rope = 56 m.

Ex. 25. The area of a circular field is 13.86 hectares. Find the cost of fencing it at the rate of Rs. 4.40 per metre.

Sol. Area = (13.86 x 10000) m^2= 138600 m^2.

∏(R^2= 138600 ⇔(R)2 = (138600 x (7/22)) ⇔ R = 210 m.

Circumference = 2∏R = (2 x (22/7) x 210) m = 1320 m.
Cost of fencing = Rs. (1320 x 4.40) = Rs. 5808.

Ex. 26. The diameter of the driving wheel of a bus is 140 em. How many revolution, per minute must the wheel make in order to keep a speed of 66 kmph ?

Sol. Distance to be covered in 1 min. = (66 X_1000)/(60) m = 1100 m.

Circumference of the wheel = (2 x (22/7) x 0.70) m = 4.4 m.

Number of revolutions per min. =(1100/4.4) = 250.

Ex, 27. A wheel makes 1000 revolutions in covering a distance of 88 km. Find the radius of the wheel.

Sol. Distance covered in one revolution =((88 X 1000)/1000)= 88m.

2∏R = 88 ⇔ 2 x (22/7) x R = 88 ⇔ R = 88 x (7/44) = 14 m.

Ex, 28. The inner circumference of a circular race track, 14 m wide, is 440 m. Find radius of the outer circle.

Sol . Let inner radius be *r* metres. Then, 2∏r = 440 ⇔ r = (440 x (7/44))= 70 m.

Radius of outer circle = (70 + 14) m = 84 m.

Ex, 29. Two concentric circles form a ring. The inner and outer circumferences of ring are (352/7) m and (518/7) m respectively. Find the width of the ring.

Sol.. Let the inner and outer radii be r and R metres.

Then 2∏*r* = (352/7) ⇔ r =((352/7) X (7/22) X (1/2))=8m.

2∏R=(528/7) ⇔ R=((528/7) X (7/22) X (1/2))= 12m.

, ', Width of the ring = (R - *r)* = (12 - 8) m = 4 m.

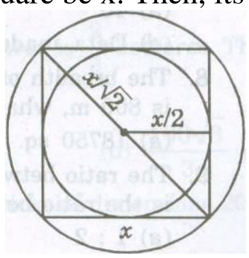
Ex, 30. A sector of 120', cut out from a circle, has an area of (66/7) sq. cm. Find the radius of the circle.

Sol. Let the radius of the circle be r cm. Then,
$(\prod(r)^2 \theta)/360=(66/7) \Leftrightarrow (22/7) X (r)^2 X(120/360)= (66/7)$
$\Leftrightarrow (r)^2=((66/7) X (7/22) X 3) \Leftrightarrow r=3.$

Hence, radius = 3 cm.

Ex, 31. Find the ratio of the areas of the incircle and circumcircle of a square.

Sol. Let the side of the square be x. Then, its diagonal = $\sqrt{2}\, x$.



Radius of incircle = (x/2)
Radius of circum circle= $(\sqrt{2}x/2) =(x/\sqrt{2})$

Required ratio = $((\prod (r)^2)/4 : (\prod(r)^2)/2) = (1/4) : 1/2) = 1 : 2.$

Ex. 32. If the radius of a circle is decreased by 50%, find the percentage decrease in its area.

Sol. Let original radius = R. New radius =(50/100) R = <u>(R/2)</u>

Original area=$\prod(R)^2$= and new area= $\prod ((R/2))^2= (\prod (R)^2)/4$
Decrease in area =$((3\prod (R)^2)/4 X (1/\prod(R)^2) X 100)$ % = 75%

25.VOLUME AND SURFACE AREA

IMPORTANT FORMULAE

I. CUBOID

Let length = 1, breadth = b and height = h units. Then, 1. Volume = (1 x b x h) cubic units.

2. Surface area= 2(lb + bh + lh) sq.units.

3. Diagonal.=$\sqrt{l^2 + b^2 + h^2}$ units

II. CUBE

Let each edge of a cube be of length a. Then,

1. Volume = a^3 cubic units.

2. Surface area = $6a^2$ sq. units.

3. Diagonal = $\sqrt{3}$ a units.

III. CYLINDER

Let radius of base = r and Height (or length) = h. Then,

1. Volume = ($\Pi r^2 h$) cubic units.

2. Curved surface area = ($2\Pi rh$). units.

3. Total surface area =$2\Pi r(h+r)$ sq. units

IV. CONE

Let radius of base = r and Height = h. Then,

1. Slant height, l =$\sqrt{h^2+r^2}$

2. Volume = (1/3) $\Pi r^2 h$ cubic units.

3. Curved surface area = (Πrl) sq. units.

4. Total surface area = ($\Pi rl + \Pi r^2$) sq. units.

V. SPHERE

Let the radius of the sphere be r. Then,

1. Volume = (4/3)Πr3 cubic units.

2. Surface area = ($4\Pi r^2$) sq. units.

VI. HEMISPHERE

Let the radius of a hemisphere be r. Then,

1. Volume = (2/3)Πr^3 cubic units.

2. Curved surface area = ($2\Pi r^2$) sq. units.

3. Total surface area = ($3\Pi r^2$) units.

Remember: 1 litre = 1000 cm3.

SOLVED EXAMPLES

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Ex. 1. Find the volume and surface area of a cuboid 16 m long, 14 m broad and 7 m high.

Sol. Volume = (16 x 14 x 7) m3 = 1568 m3.

1 Surface area = [2 (16 x 14 + 14 x 7 + 16 x 7)] cm^2 = (2 x 434) cm^2 = 868 cm^2.

Ex. 2. Find the length of the longest pole that can be placed in a room 12 m long 8m broad and 9m high.

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Sol. Length of longest pole = Length of the diagonal of the room

= √(12^2+8^2+9^2= .√(289)= 17 m.

Ex. 3. Tbe volume of a wall, 5 times as high as it is broad and 8 times as long as it is high, is 12.8 cu. metres. Find the breadth of the wall.

Sol. Let the breadth of the wall be x metres.

Then, Height = 5x metres and Length = 40x metres.

∴x * 5x * 40x = 12.8 ⇔ x^3=12.8/200 = 128/2000 = 64/1000

So, x = (4/10) m =((4/10)*100)cm = 40 cm

Ex. 4. Find the number of bricks, each measuring 24 cm x 12 cm x 8 cm, required to construct a wall 24 m long, 8m high and 60 cm thick, if 10% of the wall is filled with mortar?

Sol. Volume of the wall = (2400 x 800 x 60) cu. cm.

Volume of bricks = 90% of the volume of the wall

=((90/100)*2400 *800 * 60)cu.cm.

Volume of 1 brick = (24 x 12 x 8) cu. cm.

∴Number of bricks=(90/100)*(2400*800*60)/(24*12*8)=45000.

Ex. 5. Water flows into a tank 200 m x 160 m througb a rectangular pipe of 1.5m x 1.25 m @ 20 kmph . In what time (in minutes) will the water rise by 2 metres?

Sol. Volume required in the tank = (200 x 150 x 2) m^3 = 60000 m^3.

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Length of water column flown in1 min =(20*1000)/60 m =1000/3 m

Volume flown per minute = 1.5 * 1.25 * (1000/3) m^3 = 625 m^3.

∴ Required time = (60000/625)min = 96min

Ex. 6. Tbe dimensions of an open box are 50 cm, 40 cm and 23 cm. Its thickness is 2 cm. If 1 cubic cm of metal used in the box weighs 0.5 gms, find the weight of the box.

Sol. Volume of the metal used in the box = External Volume - Internal Volume

= [(50 * 40 * 23) - (44 * 34 * 20)]cm^3

= 16080 cm^3

∴ Weight of the metal =((16080*0.5)/1000) kg = 8.04 kg.

Ex. 7. The diagonal of a cube is 6√3cm. Find its volume and surface area.

Sol. Let the edge of the cube be a.

∴ $\sqrt{3}a$ = 6../3 _ a = 6.

So,Volume = a^3 = (6 x 6 x 6) cm^3 = 216 cm^3.

Surface area = $6a^2$ = (6 x 6 x 6) cm^2 == 216 cm^2.

Ex. 8. The surface area of a cube is 1734 sq. cm. Find its volume.

Sol. Let the edge of the cube bea. Then,

$6a^2 = 1734 \Rightarrow a^2 = 289$ => a = 17 cm.

∴ Volume = $a^3 = (17)^3$ cm^3 = 4913 cm^3.

Ex. 9. A rectangular block 6 cm by 12 cm by 15 cm is cut up into an exact number of equal cubes. Find the least possible number of cubes.

Sol. Volume of the block = (6 x 12 x 15) cm^3 = 1080 cm^3.

Side of the largest cube = H.C.F. of 6 cm, 12 cm, 15 cm = 3 cm.

Volume of this cube = (3 x 3 x 3) cm^3 = 27 cm^3.

Number of cubes = 1080/27 = 40.

Ex.l0. A cube of edge 15 cm is immersed completely in a rectangular vessel containing water . If the dimensions of the base of vessel are 20 cm x 15 cm, find the rise in water level.

Sol. Increase in volume = Volume of the cube = (15 x 15 x 15) cm^3.

∴Rise in water level = volume/area = (15 x 15 x 15)/(20 x 15) cm = 11.25 cm.

Ex. 11. Three solid cubes of sides 1 cm, 6 cm and 8 cm are melted to form a new cube. Find the surface area of the cube so formed.

Sol. Volume of new cube = $(1^3 + 6^3 + 8^3)$ cm+ = 729 cm^3.

Edge of new cube = $_3\sqrt{729}$ cm = 9 cm.

∴ Surface area of the new cube = (6 x 9 x 9) cm^2 = 486 cm^2.

Ex. 12. If each edge of a cube is increased by 50%, find the percentage increase in
Its surface area. "

Sol. Let original length of each edge = a.

Then, original surface area = $6a^2$.

New edge = (150% of a) = (150a/100) = 3a/2

New surface area = $6x\,(3a/2)^2 = 27a^2/2$

Increase percent in surface area = $\left(\left(\frac{15a^2}{2}\right) \times \left(\frac{1}{6a^2}\right) \times 100\right)\% = 125\%$

Ex. 13. Two cubes have their volumes in the ratio 1 : 27. Find the ratio of their surface areas.

Sol. Let their edges be a and b. Then,

$a^3/b^3 = 1/27$ (or) $(a/b)^3 = (1/3)^3$ (or) $(a/b) = (1/3)$.

$\therefore$ Ratio of their surface area = $6a^2/6b^2 = a^2/b^2 = (a/b)^2 = 1/9$, i.e. 1:9.

Ex.14.Find the volume , curved surface area and the total surface area of a cylinder with diameter of base 7 cm and height 40 cm.

Sol. Volume = $\prod r^2 h = ((22/7)x(7/2)x(7/2)x40) = 1540\ cm^3$.

Curved surface area = $2\prod rh = (2x(22/7)x(7/2)x40) = 880\ cm^2$.

Total surface area = $2\prod rh + 2\prod r^2 = 2\prod r\,(h + r)$

$= (2 \times (22/7) \times (7/2) \times (40+3.5))\ cm^2$

$= 957\ cm^2$

Ex.15. If the capacity of a cylindrical tank is 1848 m^3 and the diameter of its base is 14 m, then find the depth of the tank.

Sol. Let the depth of the tank be h metres. Then,

$\prod \times 7^2 \times h = 1848 \Rightarrow h = (1848 \times (7/22) \times (1/49) = 12$ m

Ex.16. 2.2 cubic dm of lead is to be drawn into a cylindrical wire 0.50 cm **diameter. Find the length of the wire in metres.**

Sol. Let the length of the wire be h metres. Then,

$\prod\ (0.50/(2 \times 100))^2 \times h = 2.2/1000$

$\Rightarrow\ h = (\ (2.2/1000) \times (100 \times 100)/(0.25 \times 0.25) \times (7/22)\ ) = 112$ m.

Ex. 17. How many iron rods, each of length 7 m and diameter 2 cm can be made out of 0.88 cubic metre of iron?

Sol. Volume of 1 rod = ((22/7) x (1/100) x (1/100) x 7) cu.m = 11/5000 cu.m

Volume of iron = 0.88 cu. m.

Number of rods = (0.88 x 5000/11) = 400.

Ex. 18. The radii of two cylinders are in the ratio 3: 5 and their heights are in tbe ratio of 2 : 3. Find the ratio of their curved surface areas.

Sol. Let the radii of the cylinders be 3x, 5x and their heights be 2y, 3y respectively. Then

Ratio of their curved surface area = $\frac{2\prod \text{ X } 3x \text{ X } 2y}{2\prod \text{ X } 5x \text{ X } 3y}$ = 2/5 = 2.5

Ex. 19. If 1 cubic cm of cast iron weighs 21 gms, then find the eight of a cast iron pipe of length 1 metre with a bore of 3 cm and in which thickness of the metal is 1 em.

Sol. Inner radius = (3/2) cm = 1.5 cm, Outer radius = (1.5 + 1) = 2.5 cm.

∴ Volume of iron = $[\prod \text{ x } (2.5)^2 \text{ x } 100 - \prod \text{ x } (1.5)^2 \text{ x } 100]\ cm^3$

$= (22/7) \text{ x } 100 \text{ x } [(2.5)^2 - (1.5)^2]\ cm^3$

$= (8800/7)\ cm^3$

Weight of the pipe = ((8800/7) x (21/1000))kg = 26.4 kg.

Ex. 20. Find the slant height, volume, curved surface area and the whole surface area of a cone of radius 21 cm and height 28 cm.

Sol. Here, r = 21 cm and h = 28 cm.

∴ Slant height, $l = \sqrt{r^2 + h^2} = \sqrt{(21)^2 + (28)^2} = \sqrt{1225} = 35cm$

Ex. 21. Find the length of canvas 1.25 m wide required to build a conical tent of base radius 7 metres and height 24 metres.

Sol. Here, r = 7m and h = 24 m.

So,$l = \sqrt{(r^2 + h^2)} = \sqrt{(7^2 + 24^2)} = \sqrt{(625)}$ = 25 m.

Area of canvas = $\prod rl=((22/7)*7*25)m^2 = 550\ m^2$.

Length of canvas = (Area/Width) = (550/1.25) m = 440 m.

Ex. 22. The heights of two right circular cones are in the ratio 1 : 2 and the perimeters of their bases are in the ratio 3 : 4. Find the ratio of their volumes.

Sol. Let the radii of their bases be r and R and their heights be h and 2h respectively.

Then,$(2\prod r/2\prod R)=(3/4) \Rightarrow R=(4/3)r$.

∴ Ratio of volumes = $(((1/3)\prod r^2h)/((1/3)\prod(4/3r)^2(2h)))$=9 : 32.

Ex. 23. The radii of the bases of a cylinder and a cone are in the ratio of 3 : 4 and It heights are in the ratio 2 : 3. Find the ratio of their volumes.

Sol. Let the radii of the cylinder and the cone be 3r and 4r and their heights be 2h and 3h respectively.

:. $\frac{\text{Volume of cylinder}}{\text{Volume of cone}} = \frac{\prod \text{ x } (3r)^2 * 2h}{(1/3)\prod r^2 * 3h}$ = 9/8 = 9 : 8.

Ex. 24. A conical vessel, whose internal radius is 12 cm and height 50 cm, is full of

liquid. The contents are emptied into a cylindrical vessel with internal radius 10 cm. Find the height to which the liquid rises in the cylindrical vessel.

Sol. Volume of the liquid in the cylindrical vessel

= Volume of the conical vessel

= ((1/3)* (22/7)* 12 * 12 * 50))cm^3 = (22 *4 *12 * 50)/7 cm^3.

Let the height of the liquid in the vessel be h.

Then (22/7)*10*10*h =(22*4*12*50)/7 or h = (4*12*50)/100 = 24 cm

Ex. 25. Find the volume and surface area of a sphere of radius 10.5 cm.

3 3

Sol. Volume = (4/3)∏r^3 =(4/3)*(22/7)*(21/2)*(21/2)*(21/2) cm^3 = 4851 cm^3.

Surface area = 4∏r^2 =(4*(22/7)*(21/2)*(21/2)) cm^2 = 1386 cm^2

Ex. 26. If the radius of a sphere is increased by 50%, find the increase percent in volume and the increase percent in the surface area.

Sol. Let original radius = R. Then, new radius = (150/100)R=(3R/2)

Original volume = (4/3)∏R^3, New volume = (4/3)∏$(3R/2)^3$ =(9∏R^3/2)

Increase % in volume=((19/6)∏R^3)*(3/4∏R^3)*100))% = 237.5%

Original surface area =4∏R^2. New surface area = 4∏$(3R/2)^2$=9∏R^2

Increase % in surface area =(5∏R^2/4∏R^2) * 100) % = 125%.

Ex. 27. Find the number of lead balls, each 1 cm in diameter that can be a sphere of diameter 12 cm.

Sol. Volume of larger sphere = (4/3)∏*6*6*6) cm^3 = 288∏ cm^3.

Volume of 1 small lead ball = ((4/3)∏*(1/2)*(1/2)*(1/2)) cm^3 = ∏/6 cm^3.

∴ Number of lead balls = (288∏*(6/∏)) = 1728.

Ex.28.How many spherical bullets can be made out of a lead cylinder 28cm high and with radius 6 cm, each bullet being 1.5 cm in diameter ?

Sol. Volume of cylinder = (∏ x 6 x 6 x 28) cm^3 = (9∏/16) cm^3.

Number of bullet = $\frac{\text{Volume of cylinder}}{\text{Volume of each bullet}}$ = [(36 x 28)∏ x 16] /9∏ = 1792.

Ex.29.A copper sphere of diameter 18cm is drawn into a wire of diameter 4 mm Find the length of the wire.

Sol. Volume of sphere = ((4∏/3) x 9 x 9 x 9) cm^3 = 972∏ cm^3

Volume of sphere = (∏ x 0.2 x 0.2 x h) cm^3

∴ 972∏ = ∏ x (2/10) x (2/10) x h ⇒ h = (972 x 5 x 5)cm = [(972 x 5 x5)/100] m

= 243m

Ex.30.Two metallic right circular cones having their heights 4.1 cm and 4.3 cm and the radii of their bases 2.1 cm each, have been melted together and recast into a

sphere. Find the diameter of the sphere.

Sol. Volume of sphere = Volume of 2 cones

$$= \left(\frac{1}{3} \prod \text{ x } (2.10^2) \text{ x } 4.1 + \frac{1}{3} \prod \text{ x } (2.1)^2 \text{ x } 4.3 \right)$$

Let the radius of sphere be R

$\therefore (4/3)\prod R^3 = (1/3)\prod(2.1)^3$ or $R = 2.1$cm

Hence , diameter of the sphere = 4.2.cm

Ex.31.A Cone and a sphere have equal radii and equal volumes. Find the ratio of the sphere of the diameter of the sphere to the height of the cone.

Sol. Let radius of each be R and height of the cone be H.

Then, $(4/3) \prod R^3 = (1/3) \prod R^2H$ (or) $R/H = ¼$ (or) $2R/H = 2/4 = 1/2$

$\therefore$Required ratio = 1:2.

Ex.32.Find the volume , curved surface area and the total surface area of a hemisphere of radius 10.5 cm.

Sol. Volume = $(2\prod r^3/3) = ((2/3) \text{ x } (22/7) \text{ x } (21/2) \text{ x } (21/2) \text{ x } (21/2))\text{cm}^3$

$= 2425.5 \text{ cm}^3$

Curved surface area = $2\prod r^3 = (2 \text{ x } (22/7) \text{ x } (21/2) \text{ x } (21/2))\text{cm}^2$

$=693 \text{ cm}^2$

Total surface area = $3\prod r^3 = (3 \text{ x } (22/7) \text{ x } (21/2) \text{ x } (21/2))\text{cm}^2$

$= 1039.5 \text{ cm}^{2.}$

Ex.33.Hemispherical bowl of internal radius 9 cm contains a liquid. This liquid is to be filled into cylindrical shaped small bottles of diameter 3 cm and height 4 cm. How many bottles will be needed to empty the bowl ?

Sol. Volume of bowl = $((2\prod/3) \text{ x } 9 \text{ x } 9 \text{ x } 9) \text{ cm}^3 = 486\prod \text{ cm}^3$.

Volume of 1 bottle = $(\prod \text{ x } (3/2) \text{ x } (3/2) \text{ x } 4) \text{ cm}^3 = 9\prod \text{ cm}^3$

Number of bottles = $(486\prod/9\prod) = 54$.

Ex34.A Cone,a hemisphere and a cylinder stand on equal bases and have the same height.Find ratio of their volumes.

Sol. Let R be the radius of each

Height of the hemisphere = Its radius = R.

$\therefore$Height of each = R.

Ratio of volumes = $(1/3)\prod R^2 \text{ x } R : (2/3)\prod R^3 : \prod R^2 \text{ x } R = 1:2:3$

26. RACES AND GAMES

IMPORTANT FACTS

Races: A contest of speed in running, riding, driving, sailing or rowing is called race

Course: The ground or path on which contests are made is called a race course.

Starting Point: The point from which a race begins is known as a starting point.

Winning Point or Goal: The point set to bound a race is called a winning paint or a goal.

Winner: The person who first reaches the winning point is called a winner.

Dead Heat Race: If all the persons contesting a race reach the goal exactly at the same time, then the race is said to be a dead heat race.

Start: Suppose A and B are two contestants in a race. If before the start of the race, A is at the starting point and B is ahead of A by 12 metres, then we say that 'A gives B, a start of 12 metres. '
To cover a race of 100 metres in this case, A will have to cover 100 metres while B will
have to cover only (100 - 12) = 88 metres. i
In a 100 m race, 'A can give B 12 m' or 'A can give B a start of 12 m' or 'A beats 12 m' means that while A runs 100 m, B runs (100 - 12) = 88 m.

Games: 'A game of 100, means that the person among the contestants who scores 100m first is the winner.
If A scores 100 points while B scores only 80 points, then we say that 'A can give B 20 points.

SOLVED EXAMPLES :

Ex. 1. In a km race, A beats B by 28 metres or 7 seconds. Find A's time over the course.

Sol. Clearly, B covers 28 m in 7 seconds.

:. B's time over the course = $(\underline{278}$ x 1000) sec = 250 seconds.

:. A's time over the course = (250 - 7-) sec = 243 sec = 4 min. 3 sec.

Ex. 2. A runs 1 ¾ times as fast as B. if A gives B a start of 84 m, bow far must winning post be so that A and B might reach it at the same time?

Sol. Ratio of the rates of A and B = 7/4 : 1 = 7 : 4.
So, in a race of 7 m, A gains 3m over B.
:. 3 m are gained by A in a race of 7 m.

:. 84 m are gained by A in a race of (7/3 x 84) m = 196 m.

:. Winning post must be 196 m away from the starting point.

Ex. 3. A can run 1 km in 3 min. 10 sec. and B can cover the same distance in 3 min. 20 sec. By what distance can A beat B ?

Soln:Clearly, A beats B by 10 sec.

Distance covered by B in 10 sec. = $\left(\frac{1000}{200} \times 10\right)$m = 50 m.

Therefore A beats B by 50 metres.

Ex .4 . In a 100 m race, A runs at 8km per hour. If A gives B a start of 4 m and still him by 15 seconds, what is the speed of B ?

Sol: Time taken by A to cover 100 m =(60 X 60 / 8000) x 100 sec = 45 sec.
B covers (100 - 4) m = 96 m in (45 + 15) sec = 60 sec.
B's speed = $\left(\frac{96 \times 60 \times 60}{60 \times 1000}\right)$km/hr = 5.76 km/hr.

Ex. 5. A, Band C are three contestants in a km race. If A can give B a start of 40 m and A can give C a start of 64m how many metre's start can B give C ?

Sol: While A covers 1000 m, B covers (1000 - 40) m = 960 m and

C covers (1000 - 64) m or 936 m.
When B covers 960 m, C covers 936 m.

Ex 6. In a game of 80 points; A can give B 5 points and C 15 points. Then how many points B can give C in a game of 60 ?

Sol. A: B = 80 : 75, A : C = 80 : 65.
B/C = (B/ A * A/C) = (75 / 80 * 80 / 65) = 15/13 = 60 /52 = 60: 5
Therfore ,In a game of 60, B can give C 8 points.

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27. CALENDAR

IMPORTANT FACTS AND FOAMULAE

Under this heading we mainly deal with finding the day of the week on a particular given date the process of finding it lies on obtaining the number of odd days.

Odd Days : Number of days more than the complete number of weeks in a given

Period ., is the number of odd days during that period.

LeapYear: Every year which is divisible by 4 is called a leap year.

Thus each one of the years 1992, 1996, 2004, 2008, 2012, etc. is a leap year. Every 4th century is a leap year but no other century is a leap year.thus each one of 400, 800, 1200,' 1600, 2000, etc. is a leap year.
None of 1900, 2010, 2020, 2100, etc. is a leap year.

An year which is not a leap year is called an ordinary year.

(I)An ordinary year has 365 days. (II) A leap year has 366 days.

Counting of Odd Days:

i)1 ordinary year = 365 days = (52 weeks + 1 day).
:. An ordinary year has 1 odd day.
ii)1 leap year = 366 days = (52 weeks + 2 days).
:. A leap year has 2 odd days.
iii)100 years = 76 ordinary years + 24 leap years
= [(76 x 52) weeks + 76 days) + [(24 x 52) weeks + 48 days]
= 5200 weeks + 124 days = (5217 weeks + 5 days).
:. 100 years contain 5 odd days.
200 years contain 10 and therefore 3 odd days.
300 years contain 15 and therefore 1 odd day.
400 years contain (20 + 1) and therefore 0 odd day.
Similarly, each one of 800, 1200, 1600, 2000, etc. contains 0 odd days.
Remark: (7n + m) odd days, where m < 7 is equivalent to m odd days.
Thus, 8 odd days ≡ 1 odd day etc.

No of odd days	0	1	2	3	4	5	6
Day	Sun.	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.

SOLVED EXAMPLES

Ex: 1.Wbat was the day of the week on, 16th July, 1776?

Sol: 16th July, 1776 = (1775 years + Period from 1st Jan., 1776 to 16th July, 1776)
Counting of odd days :
1600 years have 0 odd day. 100 years have 5 odd days.
75 years = (18 leap years + 57 ordinary years)
= [(18 x 2) + (57 x 1)] odd days = 93 odd days
= (13 weeks + 2 days) = 2 odd days.

.. 1775 years have (0 + 5 + 2) odd days = 7 odd days = 0 odd day.
Jan. Feb. March April May June July
31 + 29 + 31 + 30 + 31 + 30 +16 = 198days
= (28 weeks + 2 days) =2days
:. . Total number of odd days = (0 + 2) = 2. Required day was 'Tuesday'.

Ex. 2. What was the day of the week on 16th August, 1947?

Sol. 15th August, 1947 = (1946 years + Period from 1st Jan., 1947 to 15th
Counting of odd days:
1600 years have 0 odd day. 300 years have 1 odd day.
47 years = (11 leap years + 36 ordinary years)
= [(11 x 2) + (36 x 1») odd days = 58 odd days = 2 odd days.
Jan. Feb. March April May June July Aug.
31 + 28 + 31 + 30 + 31 + 30 + 31 + 15
= 227 days = (32 weeks + 3 days) = 3,
Total number of odd days = (0 + 1 + 2 + 3) odd days = 6 odd days.
Hence, the required day was 'Saturday'.

Ex. 3. What was the day of the week on 16th April, 2000 ?

Sol. 16th April, 2000 = (1999 years + Period from 1st Jan., 2000 to 16thA'
Counting of odd days:
1600 years have 0 odd day. 300 years have 1 odd day.
99 years = (24 leap years + 75 ordinary years)
= [(24 x 2) + (75 x 1)] odd days = 123 odd days
= (17 weeks + 4 days) = 4 odd days.
Jan. Feb. March April
31 + 29 + 31 + 16 = 107 days = (15 weeks + 2 days) = 2 odd,

Total number of odd days = (0 + 1 + 4 + 2) odd days = 7 odd days = 0 odd day. Hence, the required day was 'Sunday'.

Ex. 4. On what dates of Jull.2004 did Monday fall?

Sol . Let us find the day on 1st July, 2004.
2000 years have 0 odd day. 3 ordinary years have 3 odd days.
Jan. Feb. March April May June July
31 + 29 + 31 + 30 + 31 + 30 + 1
= 183 days = (26 weeks + 1 day) = 1 t .
Total number of odd days = (0 + 3 + 1) odd days = 4 odd days.'
:. 1st July 2004 was 'Thursday',-,-
Thus, 1st Monday in July 2004 _as on 5th July.
Hence, during July 2004, Monday fell on 5th, 12th, 19th and 26th. .

Ex. 5. Prove that the calendar for the year 2008 will serve for the year 20ll

Sol. In order that the calendar for the year 2003 and 2014 be the same, 1st January of both the years must be on the same day of the week.
For this, the number of odd days between 31st Dec., 2002 and 31st Dec.,2013 must be the same.
We know that an ordinary year has 1 odd day and a leap year has 2 odd During this period, there are 3 leap years, namely 2004, 2008 and 2012 and 8 ordinary years.
Total number of odd days = (6 + 8) days = 0 odd day.
Hence, the calendar for 2003 will serve for the year 2014.

Ex. 6. Prove that any date in March of a year is the same day of the week corresponding date in November that year.

We will show that the number of odd days between last day of February and last day of October is zero. .

March	April	May	June	July	Aug.	Sept.	Oct.
31 +	30 +	31 +	30 +	31 +	31 +	30 +	31

= 241 days = 35 weeks = 0 odd day. ,Number of odd days during this period = O.

Thus, 1st March of an year will be the same day as 1st November of that year. Hence, the result follows.

28. CLOCKS

IMPORTANT FACTS

The Face or dial of a watch is a circle whose circumference is divided into 60 equal parts, called ***minute spaces***.
A clock has two hands, the smaller one is called *the* ***hour hand*** **or** ***short hand*** while the larger one is called the ***minute hand*** **or** ***long hand.*.**
i) In 60 minutes, the minute hand gains 55 minutes on the hour hand.
ii) In every hour, both the hands coincide once.
iii) The hands are in the same straight line when they are coincident or opposite to each other.
iv) When the two hands are at right angles, they are 15 minute spaces apart.
v)When the hand's are in opposite directions, they are 30 minute spaces apart.
vi)Angle traced by hour hand in 12 hrs = 360°.
vii)Angle traced by minute hand in 60 min. = 360°.

Too Fast **and** ***Too Slow: If*** **a** ***watch or*** **a** ***clock indicates*** **8.15,** ***when the correct time*** **, 8 is said** ***to be*** **15** ***minutes too fast.***
On the other hand, if it indicates **7.45,** ***when the correct time is*** **8,** ***it is said to be 15 minutes too slow.***

SOLVED EXAMPLES

Ex 1:Find the angle between the hour hand and the minute hand of a clock when 3.25.

Solution:angle traced by the hour hand in 12 hours = 360°
Angle traced by it in three hours 25 min (ie) 41/12 hrs=(360*41/12*12)° =102*1/2°
angle traced by minute hand in 60 min. = 360°.
Angle traced by it in 25 min. = (360 X 25)/60= 150°
Required angle = 1500 – 102*1/2°= 47*1/2°

Ex 2: At what time between 2 and 3 o'clock will the hands of a clock be together?

Solution: At 2 o'clock, the hour hand is at 2 and the minute hand is at 12, *i.e.* they are 10 min spaces apart.
To be together, the minute hand must gain 10 minutes over the hour hand.
Now, 55 minutes are gained by it in 60 min.

10 minutes will be gained in (60 x 10)/55 min. = 120/11 min.

The hands will coincide at 120/11 min. past 2.

Ex. 3. At what time between 4 and 5 o'clock will the hands of a clock be at right angle?

Sol: At 4 o'clock, the minute hand will be 20 min. spaces behind the hour hand, Now, when the two hands are at right angles, they are 15 min. spaces apart. So, they are at right angles in following two cases.
Case I. When minute hand is 15 min. spaces *behind the hour hand:*
In this case min. hand will have to gain (20 - 15) = 5 minute spaces. 55 min. spaces are gained by it in 60 min.

5 min spaces will be gained by it in 60*5/55 min=60/11min.

:. They are at right angles at 60/11min. past 4.

Case II. When the minute hand is 15 min. spaces *ahead* of the *hour* hand:
To be in this position, the minute hand will have to gain (20 + 15) = 35 minute spa'
55 min. spaces are gained in 60 min.

35 min spaces are gained in (60 x 35)/55 min =40/11

:. They are at right angles at 40/11 min. past 4.

Ex. 4. Find at what time between 8 and 9 o'clock will the hands of a clock being the same straight line but not together.

Sol: At 8 o'clock, the hour hand is at 8 and the minute hand is at 12, i.e. the two hands_ are 20 min. spaces apart.
To be in the same straight line but not together they will be 30 minute spaces apart.
So, the minute hand will have to gain (30 - 20) = 10 minute spaces over the hour

hand.
55 minute spaces are gained. in 60 min.

10 minute spaces will be gained in (60 x 10)/55 min. = 120/11min.

:. The hands will be in the same straight line but not together at 120/11 min.

Ex. 5. At what time between 5 and 6 o'clock are the hands of a clock 3minapart?

. Sol. At 5 o'clock, the minute hand is 25 min. spaces behind the hour hand.
Case I. Minute hand is 3 min. spaces *behind the hour hand.*
In this case, the minute hand has to gain' (25 - 3) = 22 minute spaces. 55 min. are gained in 60 min.

22 min. are gaineg in (60*22)/55min. = 24 min.

:. The hands will be 3 min. apart at 24 min. past 5.
Case II. Minute hand is 3 min. spaces *ahead* of *the hour hand.*
In this case, the minute hand has to gain (25 + 3) = 28 minute spaces. 55 min. are gained in 60 min.

28 min. are gained in (60 x 28_)/55=346/11

The hands will be 3 min. apart at 346/11 min. past 5.

Ex 6. Tbe minute hand of a clock overtakes the hour hand at intervals of 65 minutes of the correct time. How much a day does the clock gain or lose?

Sol: In a correct clock, the minute hand gains 55 min. spaces over the hour hand in 60 minutes.
To be together again, the minute hand must gain 60 minutes over the hour hand. 55 min. are gained in 60 min.

60 min are gained in $\frac{60}{55}$ x 60 min =720/11 min.

But, they are together after 65 min.
Gain in 65 min =720/11-65 =5/11min.

Gain in 24 hours =(5/11 * (60*24)/65)min =440/43
The clock gains 440/43 _minutes in 24 hours.

Ex. 7. A watch which gains uniformly, is 6 min. slow at 8 o'clock in the morning Sunday and it is 6 min. 48 sec. fast at 8 p.m. on following Sunday. When was it correct?

Sol. Time from 8 a.m. on Sunday to 8 p.m. on following Sunday = 7 days 12 hours = 180 hours

The watch gains (5 + 29/5) min. or 54/5 min. in 180 hrs.
Now 54/5 min. are gained in 180 hrs.

5 min. are gained in (180 x 5/54 x 5) hrs. = 83 hrs 20 min. = 3 days 11 hrs 20 min.

Watch is correct 3 days 11 hrs 20 min. after 8 a.m. of Sunday.
It will be correct at 20 min. past 7 p.m. on Wednesday.

Ex 8. A clock is set right at 6 a.m. The clock loses 16 minutes in 24 hours. What will be the true time when the clock indicates 10 p.m. on 4th day?

Sol. Time from 5 a.m. on a day to 10 p.m. on 4th day = 89 hours.
Now 23 hrs 44 min. of this clock = 24 hours of correct clock.

356/15 hrs of this clock = 24 hours of correct clock.

89 hrs of this clock = (24 x 31556 x 89) hrs of correct clock.

= 90 hrs of correct clock.
So, the correct time is 11 p.m.

Ex. 9. A clock is set right at 8 a.m. The clock gains 10 minutes in 24 hours will be the true time when the clock indicates 1 p.m. on the following day?

Sol. Time from 8 a.m. on a day 1 p.m. on the following day = 29 hours.
24 hours 10 min. of this clock = 24 hours of the correct clock.

145 /6 hrs of this clock = 24 hrs of the correct clock

29 hrs of this clock = (24 x 6/145 x 29) hrs of the correct clock

= 28 hrs 48 min. of correct clock

The correct time is 28 hrs 48 min. after 8 a.m.
This is 48 min. past 12.

29. STOCKS AND SHARES

To start a big business or an industry, a large amount of money is needed. It is beyond the capacity of one or two persons to arrange such a huge amount. However, some persons associate together to form a company. They, then, draft a proposal, issue a prospectus(in the name of company), explaining the plan of the project and invite the public to invest money in this project. They, thus, pool up the funds from the public, by assigning them **shares** of the company.

IMPORTANT FACTS AND FORMULAE

1. **Stock-capital:** The total amount needed to run the company is called the **stock-capital**
2. **Shares or stock:** The whole capital is divided into small units, called **shares** or **stock.**
   For each investment, the company issues a share-certificate, showing the value of each share and the number of shares held by a person.
   The person who subscribers in shares or stock is called a **share holder** or **stock holder.**
3. **Dividend:** The annual profit distributed among share holders is called **dividend.** Dividend is paid annually as per share or as a percentage.
4. **Face Value:** The value of a share or stock printed on the share-certificate is called its **Face Value** or **Nominal Value** or **Par Value.**
5. **Market Value:** The stocks of different companies are sold and bought in the open market through brokers at stock-exchanges. A share (or stock) is said to be:
   (i) **At premium** or **Above par**, if its market value is more than its face value.
   (ii) **At par**, if its market value is the same as its face value.
   (iii) **At discount** or **Below par**, if its market value is less than its face value.
   Thus, if a Rs.100 stock is quoted at a premium of 16, then market value of the stock = Rs. (100+16) = Rs. 116.
   Likewise, I f a Rs. 100 stock is quoted at a discount of 7, then market value of the stock = Rs. (100-7) = Rs. 93.
6. **Brokerage:** The broker's charge is called **brokerage.**
   (i) When stock is purchased, brokerage is added to the cost price.
   (ii) When stock is sold, brokerage is subtracted from the selling price.
   Remember:
   (i) The face value of a share always remains the same.
   (ii) The market value of a share changes form time to time.
   (iii) Dividend is always paid on the face value of a share.
   (iv) Number of shares held by a person

$$= \frac{\text{Total Investment}}{\text{Investment in 1 share}} = \frac{\text{Total Income}}{\text{Income from 1 share}} = \frac{\text{Total Face Value}}{\text{face Value of 1 share}}$$

Thus, by a Rs. 100, 9% stock at 120, we mean that:

(i) Face Value (N>V) of stock = Rs. 100.

(ii) Market Value (M>V) of stock = Rs. 120.

(iii) Annual dividend on 1 share = 9% of face value = 9% of Rs. 100 = Rs. 9.

(iv) An investment of Rs. 120 gives an annual income of Rs. 9.

(v) Rate of interest p.a = Annual income from an investment of Rs. 100.

$$= \left(9/120 * 100\right) \% = 7\ (1/2)\ \%.$$

SOLVED EXAMPLES

Ex. 1. Find the cost of:

(i) Rs. 7200, 8% stock at 90;

(ii) Rs. 4500, 8.5% stock at 4 premium;

(iii) Rs. 6400, 10% stock at 15 discount.

Sol. **(i)** Cost of Rs. 100 stock = Rs. 90
Cost of Rs. 7200 stock = Rs. (90/100 * 7200) = Rs. 6480.

(ii) Cost of Rs. 100 stock = Rs. (100+4)
Cost of Rs. 4500 stock = Rs. (104/100 * 4500) = Rs. 4680

(iii) Cost of Rs. 100 stock = Rs. (100-15)
Cost of Rs. 6400 stock = Rs. (85/100 * 6400) = Rs. 5440.

Ex. 2. Find the cash required to purchase Rs. 3200, 7(1/2) % stock at 107 (brokerage (1/2) %)

Sol. Cash required to purchase Rs. 100 stock = Rs (107+(1/2)) = Rs. (215/2).
Cash required to purchase Rs. 100 stock = Rs [(215/2)*(1/100)*3200] = Rs. 3440.

Ex. 3. Find the cash realised by selling Rs. 2440, 9.5% stock at 4 discount (brokerage (1/4) %)

Sol. By selling Rs. 100 stock , cash realised = Rs. [(100-4)-(1/4)] = Rs. (383/4).
By selling Rs. 2400 stock, cash realised = Rs. [(383/4)*(1/100)*2400] = Rs 2298.

Ex. 4. Find the annual income derived from Rs. 2500, 8% stock at 106.

Sol. Income from Rs. 100 stock = Rs. 8.
Income from Rs. 2500 = Rs. [(8/1000*2500) =Rs. 200.

Ex. 5. Find the annual income derived by investing Rs. 6800 in 10% stock at 136.

Sol. By investing Rs. 136, income obtained = Rs. 10.

By investing Rs. 6800, income obtained = Rs. [(10/136)*6800] = Rs. 500.

Ex. 6. Which is better investment? 7(1/2) % stock at 105 or 6(1/2) % at 94.

Sol. Let the investment in each case be Rs. (105*94).

Case I : 7(1/2) 5 stock at 105:
On investing Rs. 105, income = Rs. (15/2).
On investing Rs. (105*94), income = Rs. [(15/2)*(1/105)*105*94] = Rs 705.

Case II : 6(1/2) % stock at 94:
On investing Rs. 94, income = Rs. (13/2).
On investing Rs. (105*94), income = Rs. [(13/2)*(1/94)*105*94] = Rs. 682.5.
Clearly, the income from 7(1/2) % stock at 105 is more.
Hence, the investment in 7(1/2) % stock at 105 is better.

Ex. 7. Find the cost of 96 shares of Rs. 10 each at (3/4) discount, brokerage being (1/4) per share.

Sol. Cost of 1 share = Rs. [(10-(3/4)) + (1/4)] = Rs. (19/2).
Cost of 96 shares = Rs. [(19/2)*96] = Rs. 912.

Ex. 8. Find the income derived from 88 shares of Rs. 25 each at 5 premium, brokerage being (1/4) per share and the rate of dividend being 7(1/2) % per annum. Also, find the rate of interest on the investment.

Sol. Cost of 1 share = Rs. [25+5+1/4)] = Rs. (121/4).
Cost of 88 shares = Rs.[(121/4)*88] = Rs. 2662.
∴ Investment made = Rs. 2662.
Face value of 88 shares = Rs. (88*25) = Rs. 2200.
Dividend on Rs. 100 = (15/2).
Dividend on Rs. 2200 = Rs. [(15/20*(1/100)*2200] = Rs. 165.
∴ Income derived = Rs. 165.
Rate of interest on investment = [(165/2662)*100] = 6.2 %.

Ex. 9. A man buys Rs. 25 shares in company which pays 9 % dividend. The money invested is such that it gives 10 % on investment. At what price did he buy the shares?

Sol. Suppose he buys each share for Rs. x.
Then, [25*(9/100)] = [x*(10/100)] or x = Rs. 22.50.
Cost of each share = Rs. 22.50.

Ex. 10. A man sells Rs.5000, 12 % stock at 156 and uinvests the proceeds parity in 8 % stock at 90 and 9 % stock at 108. He hereby increases his income by Rs. 70. How much of the proceeds were invested in each stock?

Sol. S.P of Rs. 5000 stock = Rs. [(156/100)*5000] = Rs. 7800.

Income from this stock = Rs. [(12/100)*5000] = Rs. 600.
Let investment in * % stock be x and that in 9 % stock = (7800-x).

∴ [x*(8/90)] + (7800-x) * (9/108) = (600+7)

⇔ (4x/45) + [(7800-x)/12] = 670 ⇔ 16x + 117000-15x = (670*180) ⇔ x = 3600.

∴ Money invested in 8 % stock at 90 = Rs. 3600.
Money invested in 9 % at 108 = Rs. (7800-3600) = Rs. 4200.

30. PERMUTATIONS AND COMBINATIONS

IMPORTANT FACTS AND FORMULAE

Factorial Notation: Let n be a positive integer. Then, factorial n, denoted by n! is defined as:

$$n! = n(n-1)(n-2)........3.2.1.$$

Examples: (i) 5! = (5x 4 x 3 x 2 x 1) = 120; (ii) 4! = (4x3x2x1) = 24 etc.
We define, 0! = 1.

Permutations: The different arrangements of a given number of things by taking some or all at a time, are called permutations.

Ex. 1. All permutations (or arrangements) made with the letters a, b, c by taking two at a time are: **(ab, ba, ac, bc, cb).**

Ex. 2. All permutations made with the letters a,b,c, taking all at a time are:
(abc, acb, bca, cab, cba).

Number of Permutations: Number of all permutations of n things, taken r at a time, given by:

$$^{n}P_{r} = n(n-1)(n-2).....(n-r+1) = n!/(n-r)!$$

Examples: (i) $^{6}p_{2}$ = (6x5) = 30. (ii) $^{7}p_{3}$ = (7x6x5) = 210.

Cor. Number of all permutations of n things, taken all at a time = n!

An Important Result: If there are n objects of which p_1 are alike of one kind; p_2 are alike of another kind; p_3 are alike of third kind and so on and p_r are alike of rth kind, such that $(p_1+p_2+.......p_r) = n$.

Then, number of permutations of these n objects is:

$$n! \;/\; (p_1!).p_2!)......(p_r!)$$

Combinations: Each of the different groups or selections which can be formed by taking some or all of a number of objects, is called a combination.

Ex. 1. Suppose we want to select two out of three boys A, B, C. Then, possible selections are AB, BC and CA.
Note that AB and BA represent the same selection.

Ex. 2. All the combinations formed by a, b, c, taking two at a time are **ab, bc, ca.**

Ex. 3. The only combination that can be formed of three letters a, b, c taken all at a time is **abc.**

Ex. 4. Various groups of 2 out of four presons A, B, C, D are:

AB, AC, AD, BC, BD, CD.

Ex. 5. Note that ab and ba are two different permutations but they represent the same combination.

Number of Combinations: The number of all combination of n things, taken r at a time is:

$$^{n}C_{r} = n! / (r!)(n-r)! = n(n-1)(n-2).....\text{to r factors} / r!$$

Note that: $^{n}c_{r} = 1$ and $^{n}c_{0} = 1$.

An Important Result: $^{n}c_{r} = {}^{n}c_{(n-r)}$.

Example: (i) $^{11}c_{4} = (11x10x9x8)/(4x3x2x1) = 330$.

(ii) $^{16}c_{13} = {}^{16}c_{(16-13)} = 16x15x14/3! = 16x15x14/3x2x1 = 560$.

SOLVED EXAMPLES

Ex. 1. Evaluate: 30!/28!

Sol. We have, 30!/28! = 30x29x(28!)/28! = (30x29) = 870.

Ex. 2. Find the value of (i) $^{60}p_{3}$ (ii) $^{4}p_{4}$

Sol. (i) $^{60}p_{3} = 60!/(60-3)! = 60!/57! = 60x59x58x(57!)/57! = (60x59x58) = 205320$.

(ii) $^{4}p_{4} = 4! = (4x3x2x1) = 24$.

Ex. 3. Find the vale of (i) $^{10}c_{3}$ (ii) $^{100}c_{98}$ (iii) $^{50}c_{50}$

Sol. (i) $^{10}c_{3} = 10x9x8/3! = 120$.

(ii) $^{100}c_{98} = {}^{100}c_{(100-98)} = 100x99/2! = 4950$.

(iii) $^{50}c_{50} = 1$. $[^{n}c_{n} = 1]$

Ex. 4. How many words can be formed by using all letters of the word "BIHAR"

Sol. The word BIHAR contains 5 different letters.

Required number of words = $^{5}p_{5} = 5! = (5x4x3x2x1) = 120$.

Ex. 5. How many words can be formed by using all letters of the word 'DAUGHTER' so that the vowels always come together?

Sol. Given word contains 8 different letters. When the vowels AUE are always together, we may suppose them to form an entity, treated as one letter.
Then, the letters to be arranged are DGNTR (AUE).

Then 6 letters to be arranged in $^{6}p_{6} = 6! = 720$ ways.

The vowels in the group (AUE) may be arranged in 3! = 6 ways.

Required number of words = (720x6) = 4320.

Ex. 6. How many words can be formed from the letters of the word 'EXTRA' so that the vowels are never together?

Sol. The given word contains 5 different letters.
Taking the vowels EA together, we treat them as one letter.
Then, the letters to be arranged are XTR (EA).
These letters can be arranged in 4! = 24 ways.
The vowels EA may be arranged amongst themselves in 2! = 2 ways.
Number of words, each having vowels together = (24x2) = 48 ways.
Total number of words formed by using all the letters of the given words
= 5! = (5x4x3x2x1) = 120.

Number of words, each having vowels never together = (120-48) = 72.

Ex. 7. How many words can be formed from the letters of the word 'DIRECTOR'
So that the vowels are always together?

Sol. In the given word, we treat the vowels IEO as one letter.
Thus, we have DRCTR (IEO).
This group has 6 letters of which R occurs 2 times and others are different.
Number of ways of arranging these letters = 6!/2! = 360.
Now 3 vowels can be arranged among themselves in 3! = 6 ways.
Required number of ways = (360x6) = 2160.

Ex. 8. In how many ways can a cricket eleven be chosen out of a batch of 15 players ?

Sol. Required number of ways = ${}^{15}c_{11} = {}^{15}c_{(15\text{-}11)} = {}^{11}c_4$

$= 15x14x13x12/4x3x2x1 = 1365.$

Ex. 9. In how many ways, a committee of 5 members can be selected from 6 men and 5 ladies, consisting of 3 men and 2 ladies?

Sol. (3 men out 6) and (2 ladies out of 5) are to be chosen.

Required number of ways = $({}^{6}c_3 x {}^{5}c_2) = [6x5x4/3x2x1] \text{ x } [5x4/2x1] = 200.$

31. PROBABILITY
IMPORTANT FACTS AND FORMULA

1.Experiment :An operation which can produce some well-defined outcome is called an experiment

2.Random experiment: An experiment in which all possible outcome are known and the exact out put cannot be predicted in advance is called an random experiment

Eg of performing random experiment:

(i)rolling an unbiased dice

(ii)tossing a fair coin

(iii)drawing a card from a pack of well shuffled card

(iv)picking up a ball of certain color from a bag containing ball of different colors

Details:

(i)when we throw a coin. Then either a **head(h)** or a **tail (t)** appears.

(ii)a dice is a solid cube, having 6 faces ,marked 1,2,3,4,5,6 respectively when we throw a die , the outcome is the number that appear on its top face .

(iii)a pack of cards has 52 cards it has 13 cards of each suit ,namely spades, clubs ,hearts and diamonds

Cards of spades and clubs are black cards

Cards of hearts and diamonds are red cards

There are 4 honors of each suit

These are **aces ,king ,queen and jack**

These are called face cards

3.S**ample space :W**hen we perform an experiment ,then the set S of all possible outcome is called the sample space

eg of sample space:

(i)in tossing a coin ,s={h,t}

(ii)if two coin are tossed ,then s={hh,tt,ht,th}.

(iii)in rolling a die we have,s={1,2,3,4,5,6}.

4.event:Any subset of a sample space.

5.Probability of occurrence of an event.

let S be the sample space and E be the event .

then,$E \subseteq S$.

$P(E)=n(E)/n(S)$.

6.Results on Probability:

(i)$P(S) = 1$ (ii)$0 \leq P(E) \leq 1$ (iii)$P(\phi)=0$

(iv)For any event a and b, we have:

$P(a \cup b)=P(a)+P(b)-P(a \cup b)$

(v)If $\overline{A}$ denotes (not-a),then $P(\overline{A})=1-P(A)$.

SOLVED EXAMPLES

Ex 1. In a throw of a coin ,find the probability of getting a head.
sol. Here s={H,T} and E={H}.
P(E)=n(E)/n(S)=1/2

Ex2.Two unbiased coin are tossed .what is the probability of getting atmost one head?
sol.Here S={HH,HT,TH,TT}
Let Ee=event of getting one head
E={TT,HT,TH}
P(E)=n(E)/n(S)=3/4

Ex3.An unbiased die is tossed .find the probability of getting a multiple of 3
sol. Here S={1,2,3,4,5,6}
Let E be the event of getting the multiple of 3
then ,E={3,6}
P(E)=n(E)/n(S)=2/6=1/3

E**x4. I**n a simultaneous throw of pair of dice .find the probability of getting the total more than 7
sol. Here n(S)=(6*6)=36
let E=event of getting a total more than 7

={(2,6),(3,5),(3,6),(4,4),(4,5),(4,6),(5,3),(5,4),(5,5),(5,6),(6,2),(6,3),(6,4),(6,5),(6,6)}
P(E)=n(E)/n(S)=15/36=5/12.

Ex5. A bag contains 6 white and 4 black balls .2 balls are drawn at random. find the probability that they are of same colour.
Sol .let S be the sample space
Then n(S)=no of ways of drawing 2 balls out of (6+4)=10c2=(10*9)/(2*1)=45
Let E=event of getting both balls of same colour
Then n(E)=no of ways(2 balls out of six) or(2 balls out of 4)
=(^{6}c2+^{4}c2)=(6*5)/(2*1)+(4*3)/(2*1)=15+6=21
P(E)=n(E)/n(S)=21/45=7/15

Ex6.Two dice are thrown together .What is the probability that the sum of the number on the two faces is divided by 4 or 6
sol. Clearly n(S)=6*6=36
Let E be the event that the sum of the numbers on the two faces is divided by 4 or 6.Then

$E=\{(1,3),(1,5),(2,2),(2,4),(2,6),(3,1),(3,3),(3,5),(4,2),(4,4),(5,1),(5,3),(6,2),(6,6)\}$

$n(E)=14$.

Hence $p(e)=n(e)/n(s)=14/36=7/18$

Ex7.Two cards are drawn at random from a pack of 52 cards.what is the probability that either both are black or both are queen?

sol. We have $n(s)=52c2=(52*51)/(2*1)=1326$.

Let A=event of getting both black cards

B=event of getting both queens

$A\cap B$=event of getting queen of black cards

$n(A)={}^{26}c2=(26*25)/(2*1)=325$,

$n(B)={}^{4}c2=(4*3)/(2*1)=6$ and

$n(A\cap B)=2c2=1$

$P(A)=n(A)/n(S)=325/1326$;

$P(B)=n(B)/n(S)=6/1326$ and

$P(A\cap B)=n(A\cap B)/n(S)=1/1326$

$P(A\cup B)=P(A)+P(B)-P(A\cap B)=(325+6-1/1326)=330/1326=55/221$

32. TRUE DISCOUNT

IMPORTANT CONCEPTS

Suppose a man has to pay Rs. 156 after 4 years and the rate of interest is 14% per annum. Clearly, Rs. 100 at 14% will amount to Rs. 156 in 4 years. So, the payment of Rs. 100 now will clear off the debt of Rs. 156 due 4 years hence. We say that:
Sum due = Rs. 156 due 4 years hence;
Present Worth (P.W.) = Rs. 100;
True Discount (T.D.) = Rs. (156 - 100) = Rs. 56
(Sum due) - (P.W.).
We define : T.D. = Interest on P.W.
Amount = (P.W.) + (T.D.).
Interest is reckoned on P.W. and true discount is reckoned on the amount.

IMPORTANT FORMULAE

Let rate = R% per annum and Time = T years. Then,
1. P.W.=[100 x Amount /100 + (R x T)
=100 x T.D./ RxT
2. T.D.=[(P.W.) x R x T /100]
= [Amount x RxT/100 + (R x T)]
3.(S.I.)*(T.D.) /(S.I.)-(T.D.)
4. (S.I.) - (T.D.) - S.I. on T.D.

5. When the sum is put at compound interest, then
P.W. = Amount/[1 +R/100]^T

SOLVED EXAMPLES

Ex. 1. Find the present worth of Rs. 930 due 3 years hence at 8% per annum. Also find the discount.
Sol.

P.W=100 x Amount /[100 + (R x T)]

=Rs.100 x 930/100+ (8x3)
= (100x930)/124
= Rs. 750,
T.D. = (Amount) - (P.W.) = Rs. (930 - 750) = Rs. 180.

Ex. 2. The true discount on a bill due 9 months hence at 12% per annum is Rs. Find the amount of the bill and its present worth.

Sol. Let amount be Rs. x. Then,
x*R*T/100 + (R x T)
=T.D.
=>x * 12*3/ 4/[100+[12*3/4]]
=540
x= 540x109 = Rs.6540
Amount - Rs. 6540. P.W. = Rs. (6540 - 540) - Rs. 6000.

Ex. 3. The true discount on a certain sum of money due 3 years hence is Rb. 250 and the simple interest on the same sum for the same time and at the same rate is Rs. 375. Find the sum and the rate percent.

Sol. T.D. = Rs. 250 and S.I. = Rs. 375.
Sum due =S.I. xT.D./ S.I. -T.D.

=375x250/375- 250
=Rs.750.
Rate=[100*375/750*3]%=16 2/3%

Ex. 4. The difference between the simple interest and true discount on a certain sum
of money for 6 months at 12—% per annum is Rs. 25. Find the

sum.

Sol. Let the sum be Rs. x. Then,
T.D. = (x*25/2*1/2)/(100+(25/2*1/2))=x*25/4*4/425=x/17
S.I=x*25/2*1/2*1/100=x/16
x/16-x/17=25
=>17x-16x=25*16*17
=>x=6800
Hence, sum due = Rs. 6800.

Ex. 5. A bill falls due in 1 year. The creditor agrees to accept immediate payment of the half and to defer the payment of the other half for 2 years. By this arrangement
ins Rb. 40. What is the amount of the bill, if the money be worth 12-z% ?
Sol. Let the sum be Rs. x. Then,
[x/2+(x/2*100)/100+(25/2*2)]-[(x*100)/(100+25/2*1]

=40
=>x/2+2x/5-8x/9=40
=>x=3600
Amount of the bill - Rs. 3600.

33. BANKER'S DISCOUNT

IMPORTANT CONCEPTS

Banker's Discount : Suppose a merchant A buys goods worth, say Rs. 10,000 from another merchant B at a credit of say 5 months. Then, B prepares a bill, called the bill of exchange. A signs this bill and allows B to withdraw the amount from his bank account after exactly 5 months.
The date exactly after 5 months is called nominally due date. Three days (known as grace days) are added to it to get a date, known as legally due date.

Suppose B wants to have the money before the legally due date. Then he can have the money from the banker or a broker, who deducts S.I. on the face value (i.e., Rs. 10,000 in this case) for the period from the date on which the bill was discounted (i.e., paid by the banker) and the legally due date. This amount is known as Banker's Discount (B.D.) Thus, B.D. is the S.I. on the face value for the period from the date on which the bill was discounted and the legally due date.

Banker's Gain (B.G.) = (B.D.) - (T.D.) for the unexpired time.
Note : When the date of the bill is not given, grace days are not to be added.

IMPORTANT FORMULAE

1. B.D. = S.I. on bill for unexpired time.
2. B.G. = (B.D.)-(T.D.) = S.I. on T.D. $=(T.D)^2/P.W$.

3. T.D. = $\sqrt{(P.W. x B.G.)}$

4. B.D. =[(Amount *Rate *Time)/100]
5. Amount=[(B.D. x T.D.)/(B.D.-T.D.)]
6. T.D.= [(Amount x Rate x Time)/(100+(Rate*Time))]
7. T.D.=[(B.G. x 100)/(Rate x Time)]

SOLVED EXAMPLES

Ex. 1. A bill for Rs. 6000 is drawn on July 14 at 5 months. It is discounted on 5th October at 10%. Find the banker's discount, true discount, banker's gain and the money that the holder of the bill receives.

Sol.

Face value of the bill = Rs. 6000.
Date on which the bill was drawn = July 14 at 5 months. Nominally due date = December 14.
Legally due date = December 17.
Date on which the bill was discounted = October 5.
Unexpired time : Oct. Nov. Dec.
26 + 30 + 17 = 73 days =1/ 5Years

B.D. = S.I. on Rs. 6000 for 1/5 year
= Rs. (6000 x 10 x1/5 x1/100)= Rs. 120.

T.D. = Rs.[(6000 x 10 x1/5)/(100+(10*1/5))]
=Rs.(12000/102)=Rs. 117.64.

B.G. = (B.D.) - (T.D.) = Rs. (120 - 117.64) = Rs. 2.36.

Money received by the holder of the bill = Rs. (6000 - 120)
= Rs. 5880.

Ex. 2. If the true discount on a certain sum due 6 months hence at 15% is Rs. 120, what is the banker's discount on the same sum for the same time and at the same rate?

Sol. B.G. = S.I. on T.D.
= Rs.(120 x 15 x 1/2 x 1/100)
= Rs. 9.
(B.D.) - (T.D.) = Rs. 9.
B.D. = Rs. (120 + 9) = Rs. 129.

Ex. 3. The banker's discount on Rs. 1800 at 12% per annum is equal to the true discount on Rs. 1872 for the same time at the same rate. Find the time.
Sol.
S.I. on Rs. 1800 = T.D. on Rs. 1872.
P.W. of Rs. 1872 is Rs. 1800.
Rs. 72 is S.I. on Rs. 1800 at 12%.
Time =[(100 x 72)/ (12x1800)]year
1/3year = 4 months.

Ex. 4. The banker's discount and the true discount on a sum of money due 8 months hence are Rs. 120 and Rs. 110 respectively. Find the sum and the rate percent.
Sol.
Sum =[(B.D.*T.D.)/(B.D.-T.D.)]
= Rs.[(120x110)/(120-110)]
= Rs. 1320.
Since B.D. is S.I. on sum due, so S.I. on Rs. 1320 for 8 months is Rs. 120.
Rate =[(100 x120)/(1320 x 2/3)%
= 13 7/11%.

Ex. 5. The present worth of a bill due sometime hence is Rs. 1100 and the true discount on the bill is Rs. 110. Find the banker's discount and the banker's gain.
Sol. T.D. =√(P.W.*B.G)
B.G. =(T.D.)2/ P.W.
= Rs.[(110x110)/ 1100]

= Rs. 11.
B.D.= (T.D. + B.G.) = Rs. (110 + 11) = Rs. 121.

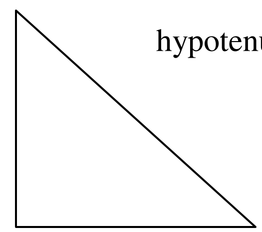
Ex. 6. The banker's discount on Rs. 1650 due a certain time hence is Rs. 165. Find the true discount and the banker's gain.
Sol.
Sum = [(B.D.xT.D.)/ (B.D.-T.D.)]
= [(B.D.xT.D.)/B.G.]
T.D./B.G. = Sum/ B.D.
=1650/165
=10/1
Thus, if B.G. is Re 1, T.D. = Rs. 10.
If B.D.is Rs. ll, T.D.=Rs. 10.
If B.D. is Rs. 165, T.D. = Rs. [(10/11)xl65]
=Rs.150
And, B.G. = Rs. (165 - 150) = Rs, 15.

Ex. 7. What rate percent does a man get for his money when in discounting a bill due 10 months hence, he deducts 10% of the amount of the bill?
Solution: Let amount of the bill = Rs.100
Money deducted =Rs.10
Money received by the holder of the bill = Rs.100-10 = Rs.90
SI on Rs.90 for 10 months = Rs.10
Rate =[(100*10)/(90*10/12)%=13 1/3%

34. HEIGHTS AND DISTANCES

IMPORTANT FACTS AND FORMULAE

1.We already know that:
In a rt.angled ΔOAB, where $\angle BOA = \theta$,
i)$\sin\theta$ = Perpendicular/Hypotenuse = AB/OB;
ii)$\cos\theta$ = Base/Hypotenuse = OA/OB;
iii)$\tan\theta$ = Perpendicular/Base = AB/OA;
iv)$\text{cosec}\,\theta = 1/\sin\theta$ = OB/AB;
v)$\sec\theta = 1/\cos\theta$ = OB/OA;
vi)$\cot\theta = 1/\tan\theta$ = OA/AB.



2. **Trigonometrical identities:**
i)$\sin^2\theta + \cos^2\theta = 1$.
ii)$1+\tan^2\theta = \sec^2\theta$
iii)$1+\cot^2\theta = \text{cosec}^2\theta$

3. **Values of T-ratios:-**

θ	0	30°	45°	60°	90°
Sin θ	0	½	$1/\sqrt{2}$	$\sqrt{3}/2$	1
Cos θ	1	$\sqrt{3}/2$	$1/\sqrt{2}$	½	0
Tan θ	0	$1/\sqrt{3}$	1	$\sqrt{3}$	Not defined

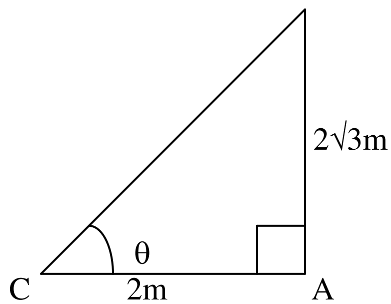
4. **Angle of Elevation:** Suppose a man from a point O looks up an object P, placed above the level of his eye. Then, the angle which the line of sight makes with the horizontal through O, is called the angle of elevation of P as seen from O.

$\therefore$ Angle of elevation of P from O = $\angle AOP$.

5. **Angle of Depression**: Suppose a man from a point O looks down at an object P, placed below the level of his eye, then the angle which the line of sight makes with the horizontal through O, is called the angle of depression of P as seen from O.

SOLVED EXAMPLES

Ex.1.If the height of a pole is 2√3 metres and the length of its shadow is 2 metres, find the angle of elevation of the sun.



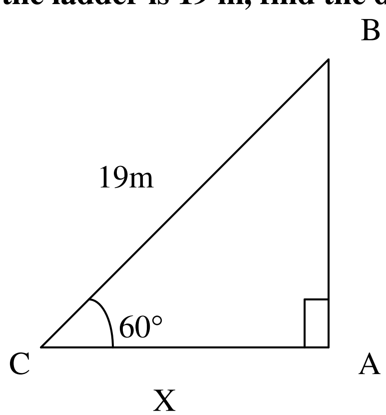
Sol. Let AB be the pole and AC be its shadow.
Let angle of elevation, ∠ACB=θ.
Then, AB = 2√3 m AC = 2 m.

Tan θ = AB/AC = 2√3/2 = √3 ⇒ θ = 60°

So, the angle of elevation is 60°

Ex.2. A ladder leaning against a wall makes an angle of 60° with the ground. If the length of the ladder is 19 m, find the distance of the foot of the ladder from the wall.

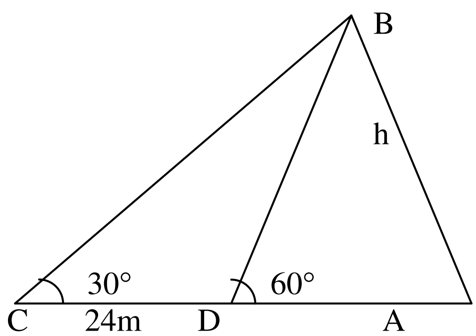
2)



Sol. Let AB be the wall and BC be the ladder.
Then, ∠ACB = 60° and BC = 19 m.
Let AC = x metres
AC/BC = cos 60° ⇒ x/19 = ½ ⇒ x=19/2 = 9.5

$\therefore$ Distance of the foot of the ladder from the wall = 9.5 m

Ex.3. The angle of elevation of the top of a tower at a point on the ground is 30°. On walking 24 m towards the tower, the angle of elevation becomes 60°. Find the height of the tower.



Sol. Let AB be the tower and C and D be the points of observation. Then,

$AB/AD = \tan 60° = \sqrt{3}$ => $AD = AB/\sqrt{3} = h/\sqrt{3}$

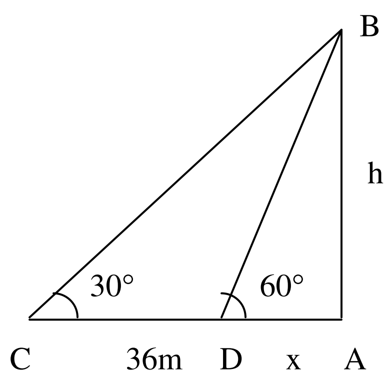
$AB/AC = \tan 30° = 1/\sqrt{3}$ $AC = AB \times \sqrt{3} = h\sqrt{3}$

$CD = (AC-AD) = (h\sqrt{3} - h/\sqrt{3})$

$h\sqrt{3} - h/\sqrt{3} = 24$ => $h = 12\sqrt{3} = (12 \times 1.73) = 20.76$

Hence, the height of the tower is 20.76 m.

Ex.4. A man standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60°. When he retires 36 m from the bank, he finds the angle to be 30°. Find the breadth of the river.



Sol. Let AB be the tree and AC be the river. Let C and D be the two positions of the man. Then,

$\angle ACB=60°$, $\angle ADB=30°$ and CD=36 m.

Let AB=h metres and AC=x metres.

Then, AD=(36+x)metres.

$AB/AD=\tan 30°=1/\sqrt{3}$ => $h/(36+x)=1/\sqrt{3}$

$h=(36+x)/\sqrt{3}$ .....(1)

$AB/AC=\tan 60°=\sqrt{3}$ => $h/x=\sqrt{3}$

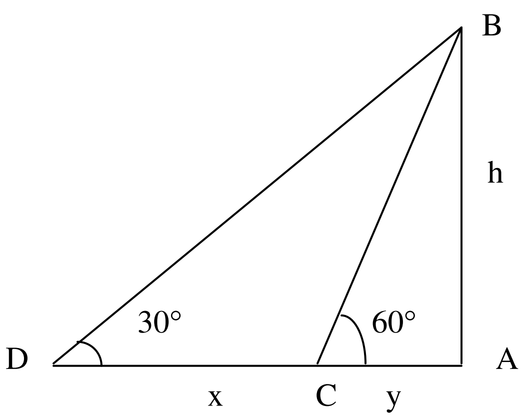
$h=\sqrt{3}x$ .....(2)

From (i) and (ii), we get:

$(36+x)/\sqrt{3}=\sqrt{3}x$ => $x=18$ m.

So, the breadth of the river = 18 m.

Ex.5. A man on the top of a tower, standing on the seashore finds that a boat coming towards him takes 10 minutes for the angle of depression to change from 30° to 60°. Find the time taken by the boat to reach the shore from this position.



Sol. Let AB be the tower and C and D be the two positions of the boat.

Let AB=h, CD=x and AD=y.

$h/y=\tan 60°=\sqrt{3}$ => $y=h/\sqrt{3}$

$h/(x+y)=\tan 30°$ = $1/\sqrt{3}$ => $x+y=\sqrt{3}h$

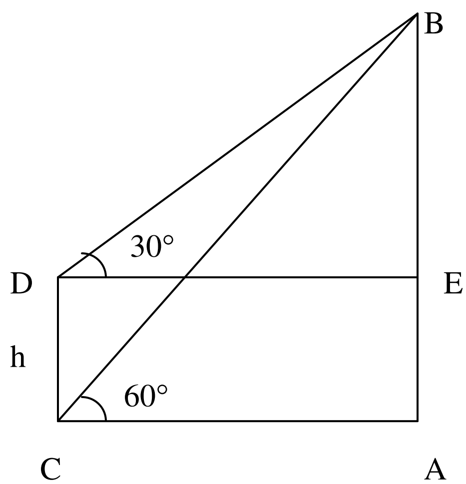
$x=(x+y)-y=(\sqrt{3}h-h/\sqrt{3})=2h/\sqrt{3}$

Now, $2h/\sqrt{3}$ is covered in 10 min.

$h/\sqrt{3}$ will be covered in $(10\times(\sqrt{3}/2h)\times(h/\sqrt{3}))=5$ min

Hence, required time = 5 minutes.

Ex 6. There are two temples, one on each bank of a river, just opposite to each other. One temple is 54 m high. From the top of this temple, the angles of depression of the top and the foot of the other temple are 30° and 60° respectively. Find the width of the river and the height of the other temple.



Sol. Let AB and CD be the two temples and AC be the river.
Then, AB = 54 m.
Let AC = x metres and CD=h metres.

$\angle ACB=60°$, $\angle EDB=30°$
$AB/AC=\tan 60°=\sqrt{3}$
$AC=AB/\sqrt{3}=54/\sqrt{3}=(54/\sqrt{3}\times\sqrt{3}/\sqrt{3})=18m$
$DE=AC=18\sqrt{3}$
$BE/DE=\tan 30°=1/\sqrt{3}$
$BE=(18\sqrt{3}\times 1/\sqrt{3})=18$ m
$CD=AE=AB-BE=(54-18)$ m = 36 m.
So, Width of the river = AC = $18\sqrt{3}$ m$=18\times1.73$ m$=31.14$m
Height of the other temple = CD= 18 m.

36. TABULATION

This section comprises of questions in which certain data regarding common disciplines as production over a period of a few years: imports, exports, incomes of employees in a factory, students applying for and qualifying a certain field of study etc. are given in the form of a table. The candidate is required to understand the given information and thereafter answer the given questions on the basis of comparative analysis of the data.

Thus, here the data collected by the investigator are arranged in a systematic form in a table called the *tabular form.* In order to avoid some heads again and again, tables are made consisting of horizontal lines called *rows* and vertical lines called *columns* with distinctive heads, known as *captions.* Units of measurements are given with the captions.

SOLVED EXAMPLES

The following table gives the sales of batteries manufactured by a *company* lit *the years.* Study *the table and answer the questions that follow:*

(S.B.I.P.O. 1998)

NUMBER OF DIFFERENT TYPES OF BATTERIES SOLD BY A COMPANY OVER THE YEARS (NUMBERS _N THOUSANDS)

TYPES OF BATTERIES

Year	4AH	7AH	32AH	35AH	55AH	T0TAL
1992	75	144	114	102	108	543
1993	90	126	102	84	426	528
1994	96	114	75	105	135	525
1995	105	90	150	90	75	510
1996	90	75	135	75	90	465
1997	105	60	165	45	120	495
1998	115	85	160	100	145	605

1. The total sales of all the seven years is the maximum for which battery ?

(a) 4AH *(b)* 7AH *(c)* 32AH *(d)* 35AH *(e)* 55AH

2. What is the difference in the number of 35AH batteries sold in 1993 and 1997 ?

(a) 24000 *(b)* 28000 *(c)* 35000 (d) 39000 *(e)* 42000

3. The percentage of 4AH batteries sold to the total number of batteries sold was maximum in the year:

(a) 1994 . *(b)* 1995 *(c)* 1996 *(d)* 1997 *(e)* 1998

4. In the case of which battery there was a continuous decrease in sales from 1992 to 1997 ?

(8) 4AH *(b)* 7 AH *(c)* 32AH (d) 35AH *(e)* 55AH

5. What was the approximate percentage increase in the sales of 55AH batteries in 1998 compared to that in 1992 ?

(a) 28% *(b)* 31% (c) 33% (d)34% (e)37%

Sol. 1. *(c)* : The total sales (in thousands) of all the seven years for various batteries are:

For 4AH = 75 + 90 + 96 + 105 + 90 + 105 + 115 = 676

For 7AH = 144 + 126 + 114 + 90 + 75 + 60 + 85 = 694

For 32AH = 114 + 102 + 75 + 150 + 135 + 165 + 160 = 901

For 35 AH= 102 + 84 + 105 + 90 + 75 + 45 + 100 = 601

For 55 AH= 108 + 126 + 135 + 75 + 90 + 120 + 145 = 799.

Clearly, sales are maximum in case of *32AH* batteries.

2. *(d* : Required difference = [(84 - 45) x 1000] = 39000.

3. *(d)* : The percentages of sales of 4AH batteries to the total sales in different years are:

For 1992 =(75*100/543)%=13.81%

For 1993=(90*100)/528%=17.05%

For 1994=(96*100/465)%=19.35%

For 1995=(105*100/495)%=20.59%

For 1996=(96*100/465)%=19.35%

For 1997=(105*100/495)%=21.21%

For 1998=(115*100/605)%=19.01%

Clearly, the percentage is maximum in 1997.

4. (b) : From the table it is clear that the sales of 7AH batteries have

been decreasing continuously from 1992 to 1997.

5. (d) : Required Percentage =(145-108)/108)*100 %=34.26%=34%.

Ex 2: Study the following table carefully and answer these questions:

NUMBER OF CANDIDATES APPEARED AND QUALIFIED IN A COMPETITIVE EXAMINATION FROM DIFFERENT STATES OVER THE YEAR

	1997		1998		1999		2000		2001	
	App.	Qual.	App.	Qal.	App.	Qual.	App.	Qual.	App.	Qual
M	5200	720	8500	980	7400	850	6800	775	9500	1125
N	7500	840	9200	1050	8450	920	9200	980	8800	1020
P	6400	780	8800	1020	7800	890	8750	1010	9750	1250
Q	8100	950	9500	1240	8700	980	9700	1200	8950	995
R	7800	870	7600	940	9800	1350	7600	945	7990	885

1. Combining the states P and Q, together in 1998, what is the percentage of the candidates qualified to that of the canditates appeared?
(8) 10.87% *(b)* 11.49% *(c)* 12.35% (d) 12.54% *(e)* 13.50%

2.The percentage of the total number of qualified candidates to the total number appeared candidates among all the five states in 1999 is :
(a) 11.49%*(b)* 11.84% *(c)*- 12.21% (d) 12.57%*(e)* 12.7a1

3. What is the percentage of candidates qualified from State N for all the years together, over the candidates appeared from State N during all the years together?
(a) 12.36% *(b)* 12.16% *(c)* 11.47% (d) 11.15%(e)None of these

4. What is the average of candidates who appeared from State Q during the given yeas?
(8) 8700 *(b)* 8760 *(c)* 8810 (d) 8920 *(e)* 8990

5 . In which of the given years the number of candidates appeared from State P has maximum percentage of qualified candidates?
(8) 1997 *(b)* 1998 *(c)* 1999 (d) 2000 (e) 2001

6. Total number of candidates qualified from all the states together in 1997

is approximately what percentage of the total number of candidates qualified from all the states together in 1998 ?

(8) 72% *(b)* 77% *(c)* 80% (d) 83% *(e)* 86%

Sol.1.(c)Required Percentage=$\frac{(1020+1240)}{(8800+9500)}$ *100%=(2260*100)/18300%

=12.35%

Required Percentage= $\frac{(850+920+890+980+1350)}{(7400+8450+7800+8700+9800)}$ *100%

=(4990*100)/42150%

=11.84%

(e) : Required Percentage=(84-+1050+920+980+1020)/(7500+9200+8450+9200+8800)*100%

=(4810*100)/43150* %

=11.15%

4. (e) Required average =(8100+9500+8700+9700+8950)/5

=44950/5

=8990

5. (e) : The percentages of candidates qualified to candidates appeared from State P during different years are:

For 1997= $\frac{780}{6400}$ * 100% =12.19%

for 1998 = $\frac{1020*100}{8800}$ %=11.59%

For 1999 = $\frac{890*100}{7800}$ %=11.41%;

For 2000 = $\frac{1010*\ 100}{8\ 750}$ % = 11.54%.

For 2001=$\frac{1250*100}{9750}$ %= 12.82%

:. Maximum percentage is for the year 2001.

6. *(c)* : Required Percentage $= \frac{(720 + 840 + 780 + 950 + 870)}{980+1050+1020+1240+940} \times 100$

$= 80\%$

Ex. 3. *The following table gives the percentage of marks obtained by* seven *students* in six , *different subjects* in an *examination. Study the table* and answer *the questions* based on *it. The numbers* in *the brackets give the maximum marks* in *each subject.*
(Bank P.O. 2003)

\ (Max. marks) Student	Maths (160)	Chemistry (130)	Physics (120)	Geography (100)	History (60)	Computer Science (40)
Ayush	90	50	90	60	70	80
Aman	100	80	80	40	80	70
Sajal	90	60	70	70	90	70
Rohit	80	65	80	80	60	60
Muskan	80	65	85	95	50	90
Tanvi	70	75	65	85	40	60
Tharun	65	35	50	77	80	80

1. What was the aggregate of marks obtained by Sajal in all the six subjects?
(a) 409 *(b)* 419 *(c)* 429 (d) 439 *(e)* 449
2. What is the overall percentage of Thrun?
(a) 52.5% *(b)* 55% *(c)* 60% (d) 63% *(e)* 64.5%
3. What are the average marks obtained by all the seven students in Physics? (rounded off to two digits after decimal)
(a) 77.26 *(b)* 89.14 *(c)* 91.37 (d) 96.11 *(e)* 103.21
4. The number of students who obtained 60% and above marks in all the subjects is :
(a) 1 *(b)* 2 *(c)* 3 (d) None *(e)* None of these
6. In which subject is the overall percentage the best?
(a) History *(b)* Maths *(c)* Physics (d) Chemistry *(e)* Geography

Sol. 1.. *(e)* : Aggregate marks obtained by Sajal

$= [(90\% \text{ of } 150) + (60\% \text{ of } 130) + (70\% \text{ of } 120) + (70\% \text{ of } 100) + (90\% \text{ of } 60) + (70\% \text{ of } 40)] = 135 + 78 + 84 + 70 + 54 + 28 = 449.$

2. *(c)* : Aggregate marks obtained by Tarun .

$= [(65\% \text{ of } 150) + (35\% \text{ of } 130) + (50\% \text{ of } 120) + (77\% \text{ of } 100) + (80\% \text{ of } 60) + (80\% \text{ of } 40)] = 97.5 + 45.5 + 60 + 77 + 48 + 32 = 360.$

Total maximum marks (of all the six subjects)

$= (150 + 130 + 120 + 100 + 60 + 40) = 600.$

Overall percentage of Tarun $= \frac{360 \times 100}{600}\ \% = 60\%.$

3. *(b)* : Average marks obtained in Physics by all the seven students

$$= \frac{1}{7}[(90\% \text{ of } 120) + (80\% \text{ of } 120) + (70\% \text{ of } 120) + (80\% \text{ of } 120) + (85\% \text{ of } 120) + (65\% \text{ of } 120) + (50\% \text{ of } 120)]$$

$$= \frac{1}{7}[(90 + 80 + 70 + 80 + 85 + 65 + 50)\% \text{ of } 120]$$

$$= \frac{1}{7}[520\% \text{ of } 120] = 89.14.$$

4. *(b)* : From the table it is clear that Sajal and Rohit have 60% or more marks in each of the six subjects.

6. *(b)* : We shall find the overall percentage (for all the seven students) with respect to each subject.

The overall percentage for any subject is equal to the average of percentages obtained by all the seven students since the maximum marks for any subject is the same for all the students.

Therefore, overall percentage for:

(i) Maths $= [\frac{1}{7}(90+100+90+80+80+70+65)]\%$

$= [\frac{1}{7}(575)]\% = 82.14\%.$

(ii) Chemistry $= [\frac{1}{7}(50 + 80 + 60 + 65 + 65 + 75 + 35)]\%$

$= [\frac{1}{7}(430)]\% = 61.43\%.$

(iii) Physics = $[\frac{1}{7}(90 + 80 + 70 + 80 + 85 + 65 + 50)]\%$

$=[\frac{1}{7}(520)]\% = 74.29\%$.

(iv) Geography = $[\frac{1}{7}(60 + 40 + 70 + 80 + 95 + 85 + 77)]\%$

$= [\frac{1}{7}(507)\}\_ = 72.43\%$.

(v) History = $[\frac{1}{7}(70 + 80 + 90+ 60 + 50 + 40 + 80)]\%$

$=\frac{1}{7}[(470)]\% = 67.14\%$.

(vi*)* Computer Science = $[1/7\ (80 + 70 + 70 + 60 + 90 + 60 + 80)]\%$

$= [\frac{1}{7}(510)]\% = 72.86\%$.

Clearly; this. percentage is highest for Maths.

ex.4. *Study the following table carefully and answer tbe questions given below:*(Bank P.O. 2001)

CLASSIFICATION OF 100 STUDENTS BASED ON THE MARKS OBTAINED BY THEM IN PHYSICS AND CHEMISTRY IN AN EXAMINATION

Marks out Of 50 Subject	40 and above	30 and Above	20 and above	10 and above	0 and above
physics	9	32	80	92	100
chemistry	4	,21	66	,. 81,	100
(aggregate Average)	7	27	73	87	100

1. The number of students scoring less than 40% marks in aggregate is :
   (a) 13 *(b)* 19 *(c)* 20 *(d)* 27 *(e)* 34
2. If at least 60% marks in Physics are required for pursuing higher studies in Physics,how many students will be eligible to pursue higher studies in Physics?
   (a) 27 *(b)* 32 *(c)* 34 *(d)41* *(e)* 68
3. What is the difference between the number of students passed with 30 as

cut-off marks in Chemistry and those passed with :JUas cut-off marks in aggregate?

(a) 3 (b) 4 (c) 5 (d) 6 (e) 7

4. The percentage of the number of students getting at least 60% marks in Chemistry over those getting at least 40% marks in aggregate, is approximately:

(a) 21% (b) 27% (c) 29% (d) 31% (e) 34%

5. If it is known that at least 23 students were eligible for a Symposium on Chemistry the minimum qualifying marks in Chemistry for eligibility to Symposium would lie in the range:

(a) 40-50 (b) 30-40 (c) 20-30 (d) Below 20

Sol. 1. (d) : We have 40% of 50 $=\left(\frac{40}{100} \times 50\right) = 20$.

:. Required number = Number of students scoring less than 20 marks in aggregate

= 100 - number of students scoring 20 and above marks in aggregate = 100 - 73 = 27.

2. (b) : We have 60% of 50 $=\left(\frac{60}{100} \times 50\right) = 30$.

:. Required number = Number of students scoring 30 and above mark in Physics = 32.

3. (d) : Required difference = (Number of students scoring 30 and above in mark in Chemistry) (Number of students scoring 30 and above marks in aggregate) = 27 - 21 = 6.

4. (c) : Number of students getting at least 60% marks in Chemistry
= Number of students getting 30 and above marks in Chemistry = 21.
Number of students getting at least 40% marks in aggregate
= Number of students getting 20 and above marks in aggregate = 73.
:. Required Percentage $= \left(\frac{21}{73} \times 100\right)\% = 28.77\% \approx 29\%$.

6. (c) : Since 66 students get 20 and above marks in Chemistry and out of these 21 students get 30 and above marks, therefore to select top 35 students in Chemistry, the qualifying marks should lie in the range 20-30.

37.BAR GRAPHS

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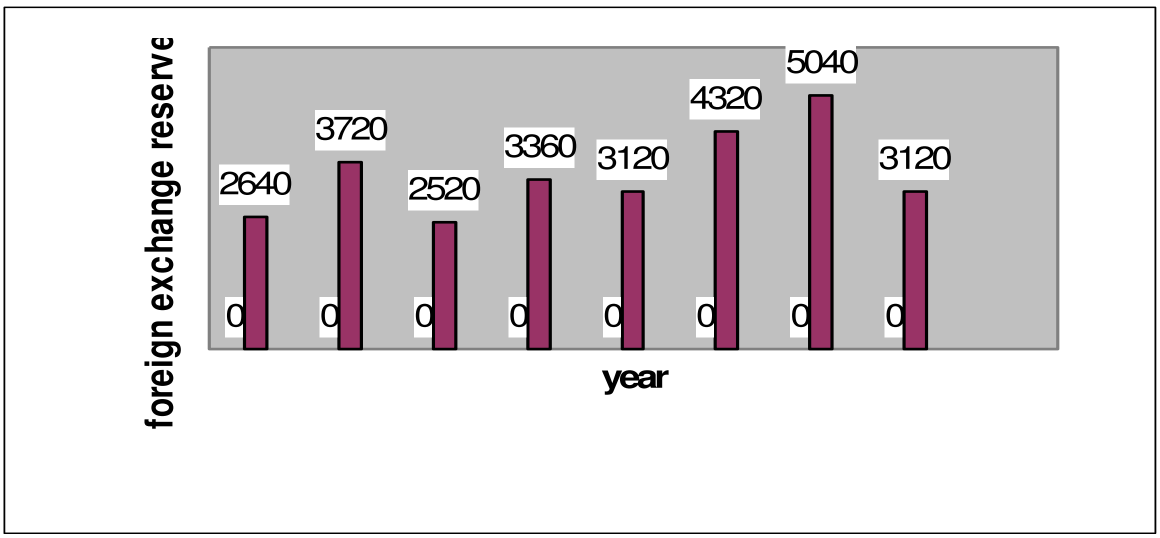
this section comprises of questions in which the data collected in a particular discipline are represented in the form of vertical or horizontal bars drawn by selecting a particular scale.one of the parameters is plotted on the horizontal axis and the other on the vertical axis . the candidate is required to understand the given information and thereafetr answer the given questions on the basis of data analysis.

1.The bar graph given below shows the foreign exchange reserves of a country (in million us$)
from 1991-92 to 1998-99 .answer the questions based
on this graph.

FOREIGN EXCHANGE RESERVES OF A COUNTRY
(IN MILLION US $)



1.The foreign exchange reserves in 1997-98 was how ,any times that in 1994-95?
(a)0.7 (b) 1.2 (c) 1.4 (d) 1.5 (e) 1.8
2.what was the percentage increase in the foreign exchange reserves in 1997-98 over 1993-94?
(a)100 (b)150 (c)200 (d)620 (e)2520

3.for which year,the percent increase of foreign exchange reserves over the previous year,is the highest?
(a)1992-93 (b)1993-94 (c)1994-95 (d)1996-97 (e)1997-98

4.the foreign exchange reserves in 1996-97 werw approximately what percent of the average foreign exchange reserves over the period under review?

(a)95% (b)110% (c)115% (d)125% (e)140%

5.the ratio of the number of years,in which the foreign exchange reserves are above the average reserves ,to those in which the reserves are below the average reserves is :

(a)2:6 (b)3:4 (c)3:5 (d)4:4 (e)5:3

Solutions

1 (d) : required ratio = 5040/3360 = 1.5

2 (a) : foreign exchange reserve in 1997-98=5040 million us $
foreign exchange reserves in 1993-94=2520 million us$
therefore increase=(5040-2520)=2520 million us $
therefore percentage increase=((2520/2520)*100)%=100%

3(a): there is an increase in foreign exchange reserves during the years 1992-93,1994-951996-97,1997-98 as compared to previous year (as shown by bar graph)
the percentage increase in reserves during these years compared to previous year are

(1) for 1992-93 =[(3720-2640)/2640*100]% =40.91%
(2) for 1994-95=[(3360-2520)/2520*100]%=33.33%
(3) for 1996-97=[(4320-3120)/3120*100]%=38.46%
(4) for 1997-98=[(5040-4320)/4320*100]%=16.67%

Clearly, the percentage increase over previous year is highest for 1992-93.

4. (d) : Average foreign exchange reserves over the given period
= [_x (2640 + 3720 + 2520 + 3360 + 3120 + 4320 + 5040 + 3120)] million US $

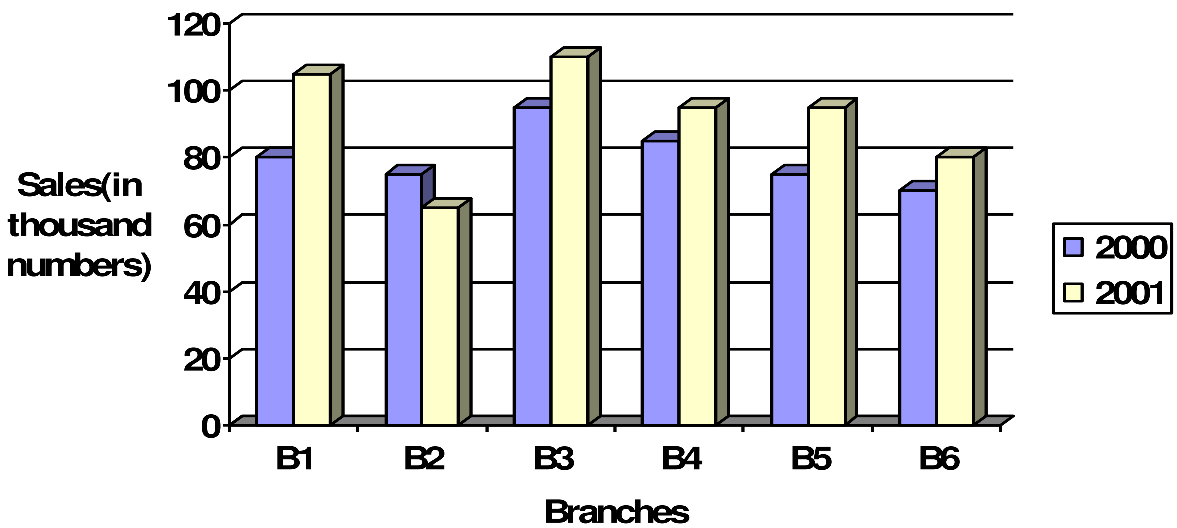
= 3480 million US $.
Foreign exchange reserves in 1996-97 = 4320 million US $. . .
Required Percentage = x 100 % = 124.14% .. 125%.
3480 .

5. *(c)* : Average foreign exchange reserves over the given period = 3480 million US $.

The country had reserves above 3480 million US $ during the years 199293, 1996-97 and 1997-98 *i.e.,* for 3 years and below 3480 million US $ during the years 1991-92, 1993-94, 1994-95, 1995-96 and 1998-99 *i.e.,* for 5 years.
Hence, required ratio = 3 : 5.

Ex. 2. The bar-graph provided on next page gives the sales of books (in thousand numbers) from six branches of a publishing company during two consecutive years 2000 and 2001. Answer the questions based on this bar-graph:

Sales of books (in thousand numbers) from six branches-B1,B2,B3,B4,B5 and B6 of a publishing company in 2000 and 2001



1.total sales of branches b1,b3 and b5 together for both the years (in thousand numbers) is:
(a)250 (b) 310 (c) 435 (d)560 (e)585

2.total sales of branch b6 for both the years is what percent of the total sales of branch b3 for both the years?
(a) 68.54% (b)71.11% (c) 73.17% (d)75.55% (e)77.26%

3.what is the average sale of all the branches (in thousand numbers) for the year 2000?
(a)73 (b)80 (c)83 (d)88 (e)96

4.what is the ratio of the total sales of branch b2 for both years to the total sales of branch b4 for both years ?
(a)2:3 (b)3:5 (c)4:5 (d)5:7 (e)7:9

5.what percent of the average sales of branchesn b1,b2 and b3 in 2001 is the average sales of branches b1,b3 and b6 in 2000?
(a)75% (b)77.5% (c)82.5% (d)85% (e)87.5%

SOLN…………………..
1.(d) total sales of branches B1,B3 and B5 for both the years (in thousand numbers)=(80+105)+(95+110)+(75+95)=560

2(c) required percentage=[(70+80)/(95+110)*100]%=(150/205*100)%=73.17%

3(b)average sales of all the six branches (in thousand numbers) for the year 2000=1/6*(80+75+95+85+75+70)=80

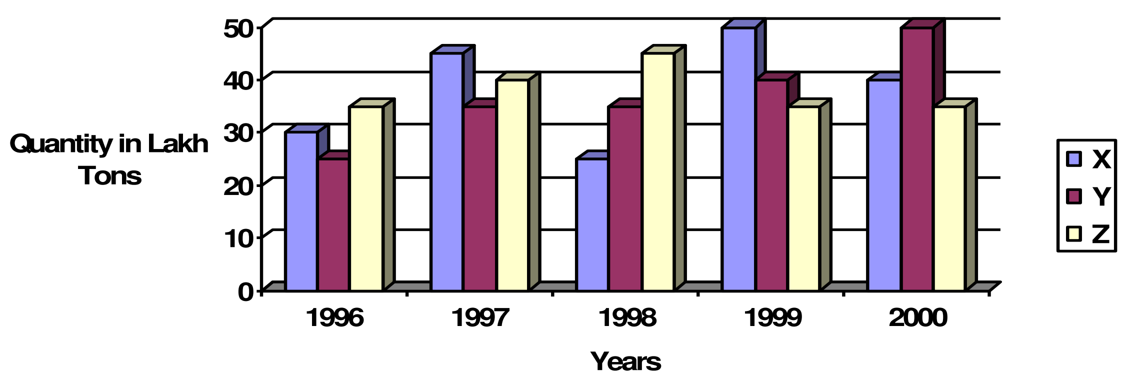
4(e) required ratio=(75+65)/(85+95)=140/180=7/9

5(e)average sales(in thousand numbers of branches B1,B3,and B6 in 2000= 1/3*(80+95+70)=245/3

average sales(in thousand numbers of branches B1,B2,and B3 in 2001=1/3*(105+65+110)=280/3

therefore required percentage=[((245/3)/(280/3))*100]%=(245/280*100)%=87.5%

Ex.3. The bar graph provided below gives the data of the production of paper (in thousand tonnes) by three different companies x,y and z over the years .study the graph and answer the questions that follow
production of paper(in laks tonnes) by three companys x,yand z over the years



1.What is the difference between the production of the company Z in 1998 and company y in 1996?
a.2 ,00,000 tons
b.20,00,000 tons
c.20,000 tons
d.2,00,00,000 tons
e.none of these

2.what is the ratio of the average production of company x in the period 1998 to 2000 to the average production of company y in the same period?
a.1:1
b.15:27
c.23:25
d.27:29

e.none of these

3.what is the percentage increase in the production of company y from1996 to 1999?
a.30%
b.45%
c.50%
d.60%
e.75%

4.the avreage production of five years was maximum for which company?
a.x
b.y
c.z
d.x & y both
e.x and z both

5.for which of thw follolwing years the percentage rise / fall in production from previous year is the maximum for company y?
a.1997
b.1998
c.1999
d.2000
e.1997 & 2000

6.in which year was the percentage of production of company z to the production of company y the maximum?
a.1996
b.1997
c.1998
d.1999
e.2000
Sol: **1**(b):required difference =
[(45-25)*i,00,000]tons=20,00,000 tons

2(c):average production of company x in the period 1998-2000=[1/3*(25+50+40)]=(115/3) lakh tons

average production of company y in the period 1998-2000
[1/3*(35+40+50)]=(125/3) lakh tons

therefore req ratio=(115/3)/(125/3)=115/125=23/25

3(d):percentage increase in the production y from 1996-1999=[(40-25)/25*100]%=(15/25*100)%=60%

4(e):average production (in lakh tons)in five years for the three company's are:
for company x=[1/5*(30+45+25+50+40)]=190/5=38
for company y=[1/5*(25+35+35+40+50)]=185/5=37
for company z=[1/5*(35+40+45+35+35)]=190/5=38
therefore the average production of maximum for both the company's x and z

5(a) : Percentage change (rise/fall)in the production of Company Y in comparison to the previous year, for different years are:

For 1997 = [((32-25)/25)*100]% = 40%

For 1998 = [((35-35)/25)*100]% = 0%

For 1999 = [((40-35)/35)*100]% = 14.29%

For 2000 = [((50-40)/40)*100]% = 25%

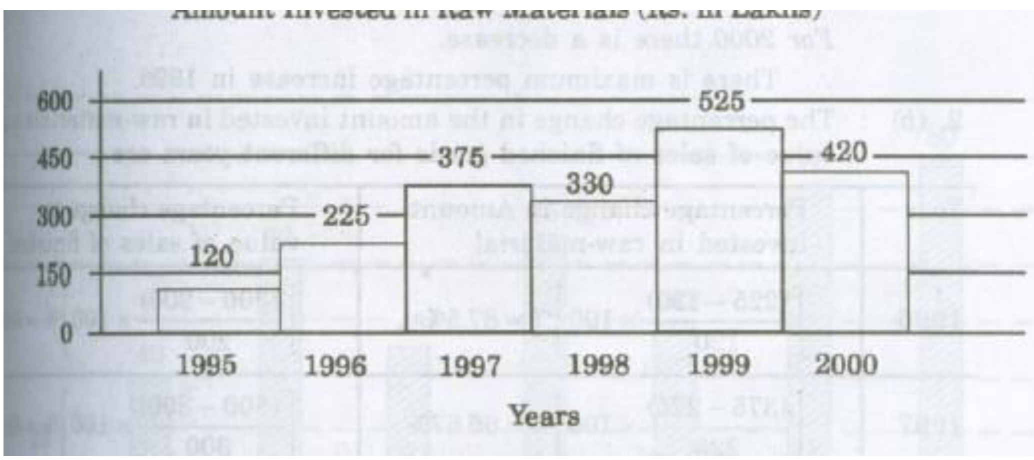
Hence, the maximum percentage rise/fall in the production of company Y is for 1997.

6(a) : The percentages of production of company z to the production of company z for various years are:
For 1996 = ((35/25)*100)%=140%; For 1997 = ((40/35)*100)% = 114.29%
For 1998 = ((45/35)*100)%=128.57%; For 1999 = ((35/40)*100)%=87.5%
For 2000 = ((35/50)*100)%=70%

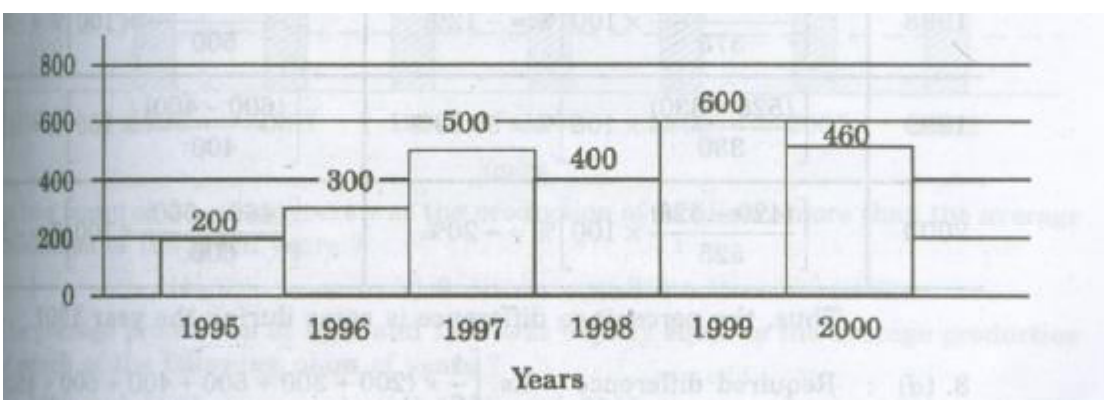
Clearly, this percentage is highest for 1996.

Ex.4.Outof the two bar graphs provided below, one shows the amounts (in Lakh Rs) invested by a Company in purchasing raw materials over the years and the other shows the values(in Lakh Rs.) of finished goods sold by the Company over the years. Study the two bar graphs and answer the questions based on them.
Amount Invested in Raw Materials and the Value of Sales of Finished Goods for a Company over the Years

Amount Invested in Raw Materials (Rs. in Lakhs)



Value of Sales of Finished Goods(Rs.in Lakhs)



1.Inwhichyear,there has been a maximum percentage increase in the amount invested inRawMaterials as compared to the previous year?
*(a)*1996 (b) 1997 *(c)* 1998 (d) 1999 *(e)* 2000
2. Inwhichyear, the percentage change (compared to the previous year) in the investment onRaw Materials is the same as that in the value of sales of finished goods?
*(a)*1996 *(b)* 1997 *(c)* 1998 (d) 1999 *(e)* 2000
3.Whatwas the difference between the average amount invested in Raw Materials during the given period and the average value of sales of finished goods during this period?
*(a)*Rs. 62.5 lakhs *(b)* Rs. 68.5 lakhs *(c)* Rs. 71.5 lakhs
(d)Rs.77.51akhs *(e)* Rs. 83.5 lakhs

4. The value of sales of finished goods in 1999 was approximately what percent of the average amount invested in Raw Materials in the years 1997,1998 and 1999?
(a) 33% (b) 37% (c) 45% (d) 49% (e) 53%

5. The maximum difference between the amount invested in Raw Materials and the value of sales of finished goods was during the year:
(a) 1995 (b) 1996 (c) 1997 (d) 1998 (e) 1999

Sol. 1. (a) : The percentage increase in the amount invested in raw-materials as compared to the previous year, for different years are:

For 1996 = [((225-120)/120)*100]% = 87.5%
For 1997 = [((375-225)/225)*100]% = 66.67%
For 1998 = [((525-330)/330)*100]% = 59.09%
For 2000 there is a decrease.

2.(b)The percentage change in the amount invested in raw-materials and in the value of sales of finished goods for different years are:

year	Percentage change in amount invested in raw-materials	Percentage change in value of sales of finished goods
1996	[((225-120)/120)*100]% = 87.5%	[((300-200)/200)*100]% = 50%
1997	[((375-225)/225)*100]% = 66.7%	[((500-300)/300)*100]% = 66.67%
1998	[((525-330)/330)*100]% = -12%	[((400-500)/500)*100]% = -20%
1999	[((525-330)/330)*100]% = 59.09%	[((600-400)/400)*100]% = 50%
2000	[((420-525)/525)*100]% = -20%	[((460-600)/600)*100]% = -23.33%

Thus the percentage difference is same during the year 1997.

3. (d) : Required difference = Rs. [(1/6)*(200+300+500+400+600+460)
- (1/6)*(120+225+375+330+525+420)] lakhs
= Rs. [(2460/6)-(1995/6)] lakhs = Rs.(410-332.5)lakhs = 77.5 lakhs.

4. (d) : Required percentage = [(600/(375+300+525))*100]% = 48.78% ≈ 49%

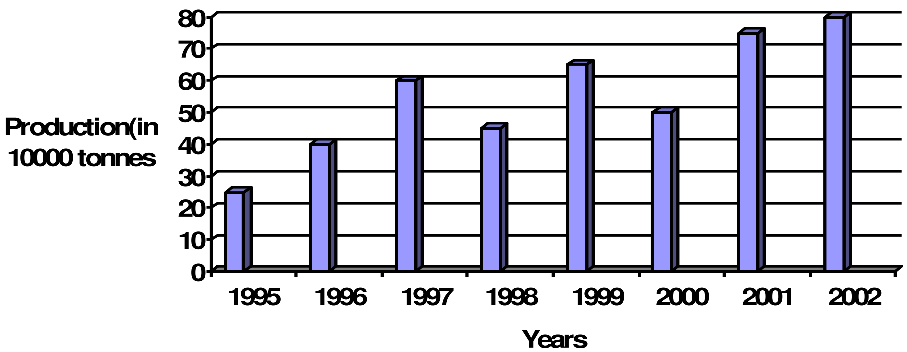
5. (c) : The difference between the amount invested in raw-material and the value of sales of finished goods for various years are :

For 1995 = Rs.(200-120)lakhs = Rs. 80 lakhs
For 1996 = Rs.(200-225)lakhs = Rs. 75 lakhs
For 1997 = Rs. (500-375)lakhs = Rs. 125 lakhs
For 1998 = Rs. (400-330)lakhs = Rs. 70 lakhs.
For 1999 = Rs. (600-525)lakhs = Rs. 75 lakhs
For 2000 = Rs. (460-420)lakhs = Rs. 40 lakhs.
Clearly, maximum difference was during 1997

EXERCISE 37

Directions(questions 1 to 5) : study the following bar-graph and answer the questions given below.
Production of fertilizers by a Company (in 10000 tonnes) over the Years

X axis=years
Y axis=Production (in 10000 tonnes)



1. **In how many of the given years was the production of fertilizers more than the average production of the given years?**

   (a)1 (b)2 (c)3 (d)4 (e)5

2. **The average production of 1996 and 1997 was exactly equal to the average production of which of the following pairs of years?**

   (a)2000 and 2001 (b)1999 and 2000 (c)1998 and 2000
   (d)1995 and 1999 (e)1995 and 2001

3.What was the percentage decline in the production of fertilizers from 1997 to 1998?

(a) $33^{1}/_{3}$% (b) 30% (c) 25% (d) 21% (e) 20%

4.In which year the percentage increase in production as compared to the previous year the maximum?

(a) 2002 (b) 2001 (c) 1999 (d) 1997 (e) 1996

38. PIE-CHARTS

IMPORTANT FACTS AND FORMULAE

The **pie-chart** or a **pie-graph** is a method of representing a given numerical data in the form of sectors of a circle.
The sectors of the circle are constructed in such a way that the area of each sector is proportional to the corresponding value of the component of the data.
From geometry, we know that the area of a circle is proportional to the central angle.
So, the central angle of each sector must be proportional to the corresponding value of the component.
Since the sum of all the central angle is 360°, we have

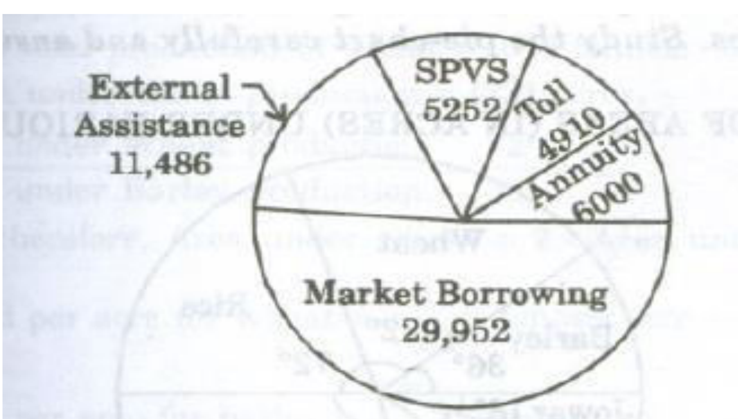
$$\text{Central angle of the component} = \left|\frac{\text{Value of the component}}{\text{Total value}} * 360\right|^{\circ}$$

SOLVED EXAMPLES

The procedure of solving problems based on pie-charts will be clear from the following solved examples.

Example 1. The following pie-chart shows the sources of funds to be collected by the National Highways Authority of India (NHAI) for its Phase II projects. Study the pie-chart and answer the questions that follow.

SOURCES OF FUNDS TO BE ARRANGED BY NHAI FOR PHASE II PROJECTS (IN CRORES RS.)



Total funds to be arranged for Projects (Phase II) =Rs.57,600 crores.

1. Near about 20% of the funds are to be arranged through:

(a) SPVS (b) External Assistance

(c) Annuity (d) Market Borrowing

2. The central angle corresponding to Market Borrowing is:

(a) 52° (b) 137.8°

(c) 187.2° (d) 192.4°

3. The approximate ratio of the funds to be arranged through Toll and that through Market Borrowing is:

(a) 2:9 (b) 1:6

(c) 3:11 (d) 2:5

4. If NHAI could receive a total of Rs. 9695 crores as External Assistance, by what percent (approximately) should it increase the Market Borrowings to arrange for the shortage of funds ?

(a) 4.5% (b) 7.5%

(c) 6% (d) 8%

5.If the toll is to be collected through an outsourced agency by allowing a maximum 10% commission, how much amount should be permitted to be collected by the outsourced agency, so that the project is supported with Rs. 4910 crores ?

(a) Rs.6213 crores (b) Rs. 5827 crores

(c) Rs. 5401 crores (d) Rs. 5216 crores

SOLUTION

1. (b): 20% of the total funds to be arranged = Rs.(20% of 57600) crores
= Rs.11520 crores Rs.11486 crores.

2. (c):Central angle corresponding to Market Borrowing = $\frac{29952}{57600} * 360°$
= 187.2°

3. (b):
Required ratio = $\frac{4910}{29952} = \frac{1}{6.1} = \frac{1}{6}$

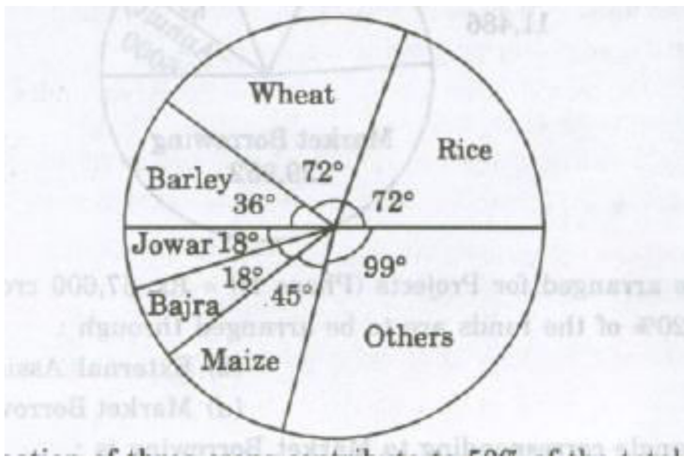
4. (c):Shortage of funds arranged through External Assistance
=Rs.(11486-9695) crores =Rs. 1791 crores.
therefore, Increase required in Market Borrowings =Rs. 1791 crores.

Percentage increase required = $\left| \frac{1791}{29952} * 100 \right|$ % = 5.98 % = 6%

5. (c):Amount permitted = (Funds required from Toll for projects of Phase II) +
(10 % of these funds)
=Rs. 4910 crores + Rs. (10% of 4910) crores
=Rs. (4910 + 491) crores = Rs. 5401 crores.

Example 2. The pie-chart provided below gives the distribution of land (in a village) under various food crops. Study the pie-chart carefully and answer the questions that follow.

DISTRIBUTION OF AREAS (IN ACRES) UNDER VARIOUS FOOD CROPS



1.Which combination of three crops contribute to 50% of the total area under the food crops ?
(a) Wheat, Barley and Jowar (b)Rice, Wheat and Jowar
(c) Rice, Wheat and Barley (d)Bajra, Maize and Rice

2.If the total area under jowar was 1.5 million acres, then what was the area (in million acres) under rice?
(a)6 (b)7.5 (c)9 (d)4.5

3.If the production of wheat is 6 times that of barley, then what is the ratio between the yield per acre of wheat and barley ?
(a) 3:2 (b) 3:1 (c) 12:1 (d) 2:3

4.If the yield per acre of rice was 50% more than that of barley, then the production of barley is what percent of that of rice ?
(a)30% (b)33 % (c)35% (d)36%

5.If the total area goes up by 5%, and the area under wheat production goes up by 12% , then what will be the angle for wheat in the new pie-chart ?
(a) 62.4° (b) 76.8° (c) 80.6° (d) 84.2°

SOLUTIONS

1.(c):The total of the central angles corresponding to the three crops which cover 50% of the total area ,should be 180°.Now, the total of the central angles for the given combinations are:

(i) Wheat,Barley and jowar = (72°+36°+18°)=126°
(ii) Rice,Wheat and Jowar = (72°+72°+18°)=162°
(iii) Rice,Wheat and Barley = (72°+72°+36°)=180°
(iv)Bajra,Maize and Rice = (18°+45°+72°) = 135°
Clearly:(iii) is the required combination.

2.(a): The area under any of the food crops is proportional to the angle corresponding to that crop.
Let the area under the rice production be x million acres.
Then, 18:72 = 1.5:x ⇒ x=(72*15/18)=6
Thus, the area under rice production be = 6 million acres.

3.(b): Let the total production of barley be T tones and let Z acres of land be put under barley production.
Then, the total production of wheat =(6T) tones.
Also,area under wheat production = (2Z) acres.

$$\therefore \frac{\text{Area Under Wheat Production}}{\text{Area Under Barley Production}} = \frac{72°}{36°} = 2$$

And therefore,Area under wheat = 2*Area under Barley = (2Z)acres

Now, yield per acre for wheat = (6T/2Z) tones/acre = (3T/Z) tones/acre
And yield per acre for barley = (T/Z) tones/acre.

$$\therefore \text{Required ratio} = \frac{3T/Z}{T/Z} = 3:1.$$

4. (b):Let Z acres of land be put under barley production.

$$\text{Then,} \quad \frac{\text{Area Under Rice Production}}{\text{Area Under Barley Production}} = \frac{72°}{36°} = 2.$$

∴ Area under rice production = 2 * area under barley production = (2Z) acres.
Now,if p tones be the yield per acre of barley then ,yield per acre of rice
=(p+50% of p) tones =(3/2 p) tones.
∴ Total production of rice = (yield per acre) * (area under production)
= (3/2 p)*2Z=(3pZ) tones.
And,Total production of barley = (pz) tones.
∴ Percentage production of barley to that rice = (pZ/3pZ *100)%= 33 1/3% .

5.(b): Initially,let t be the total area under considerations.
The area under wheat production initially was =(72/360 * t)acres = (t/5)acres.
Now,if the total area under consideration be increased by 5%,
then the new value of the total area= (105/100 t) acres.
Also,if the area under wheat production be increased by 12%,

$$\text{then the new value of area under wheat} = \left|\frac{t}{5} + (12\% \text{ of } \frac{t}{5})\right| \text{acres} = (112t/500)\text{acres}.$$

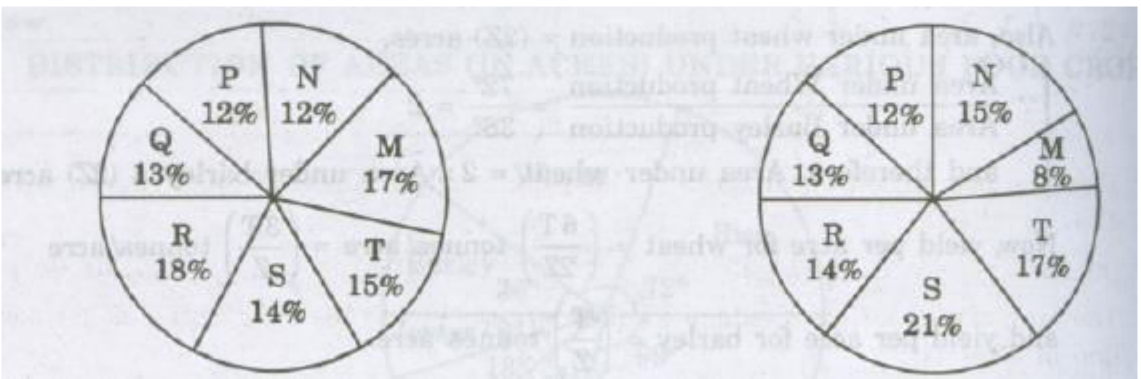
∴Central angle corresponding to wheat in the pie-chart

$$= \left[\frac{\text{Area Under Wheat (new)}}{\text{Total area (new)}} * \mathbf{360}\right]^\circ = \left|\frac{(112t/500)}{(105t/100)} * 360\right|^\circ = 76.8^\circ$$

Example 3.The following pie-charts show the distribution of students of graduate and post graduate levels in seven different institute-M,N,P,Q,R,S and T in a town.

DISTRIBUTION OF STUDENTS AT GRADUATE AND POST-GRADUATE LEVELS IN SEVEN INSTITUTES-M,N,P,Q,R,S AND T.

Total Number of students of graduate level | **Total Number of students of post graduate level**



1.How many students of institutes M and S are studying at graduate level?
(a) 7516 (b) 8463 (c) 9127 (d) 9404

2.Total number of students studying at post -graduate level from institutes N and P is:
(a) 5601 (b) 5944 (c) 6669 (d) 7004

3.What is the total number of graduate and post-graduate level students in institute R?
(a) 8320 (b) 7916 (c) 9116 (d) 8372

4 .What is the ratio between the number of students studying at post graduate and graduate levels respectively from institute S?
(a) 14:19 (b) 19:21 (c) 17:21 (d) 19:14

5.What is the ratio between the number of students studying post graduate level from institute S and the number of students studying at graduate level from institute Q?
(a) 13:19 (b) 21:13 (c) 13:8 (d)19:13

SOLUTION

1.(b):Students of institute M at graduate level = 17% of 27300 = 4641.
Students of institute S at graduate level = 14% of 27300 = 3822
$\therefore$Total number students at graduate level in institutes M and S = 4641+3822=8463

2.(c):Required number = (15% of 24700) + (12% of 24700) = 3705 + 2964 = 6669.

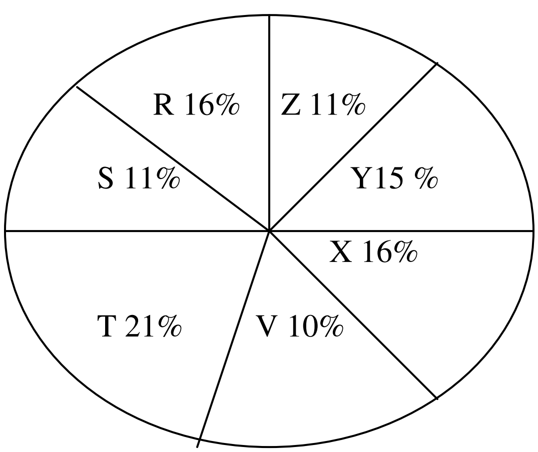
3.(d):Required number = (18% of 27300) + (14% of 24700) = 4914 + 3458 = 8372.

4.(d):Required ratio = $\frac{(21\% \text{ of } 24700)}{(14\% \text{ of } 27300)} = \frac{21 * 24700}{14 * 27300} = \frac{19}{14}$

5.(d):Required ratio = $\frac{(21\% \text{ of } 24700)}{(13\% \text{ of } 27300)} = \frac{21 * 24700}{13 * 27300} = \frac{19}{13}$

Example 4.Study the following pie-chart and the table and the answer the questions based on them.

PROPORTION OF POPULATION OF SEVEN VILLAGES IN 1997



village	% population below poverty
X	38
Y	52
Z	42
R	51
S	49
T	46
V	58

1.Find the population of villages S if the population of village X below poverty line in 1997 is 12160.
(a). 18500 (b) 20500 (c) 22000 (d) 26000

2.The ratio of the population of the village T below poverty line to that of village Z below poverty line in 1997 is:
(a) 11:23 (b) 13:11 (c) 23:11 (d) 11:13

3.If the population of village R in 1997 is 32000,then what will be the population of village Y below poverty line in that year?
(a) 14100 (b)15600 (c) 16500 (d) 17000

4.If in 1998, the population of villages Y and V increases by 10% each and the percentage of population below poverty line remains unchanged for all the villages, then find the population of village V below poverty line in 1998,given that the population of village Y in 1997 was 30000.
(a) 11250 (b) 12760 (c) 13140 (d) 13780

5.If in 1998,the population of village R increases by 10% while that of village Z reduces by 5% compared that in 1997 and the percentage of population below poverty line remains unchanged for all the village,then find the approximate ratio of population of village R below poverty line for the year 1999.
(a) 2:1 (b) 3:2 (c) 4:3 (d) 5:4

SOLUTION

1.(c):Let the population of village X be x

Then,38% of x=12160 $\Rightarrow$ $x = \frac{12160 * 100}{38} = 3200$

Now ,if s be the population village S,then

$16:11 = 32000 : s \quad \Rightarrow s = \frac{11 * 32000}{16} = 22000.$

2.(c): Let N be the total population of all the seven villages.

Then ,population of village T below poverty line = 46% of (21% of N) and population of village Z below poverty line = 42% of (11% of N)

$$\therefore \text{Required ratio} = \frac{46\% \text{ of } (21\% \text{ of N})}{42\% \text{ of } (11\% \text{ of N})} = \frac{46 * 21}{42 * 11} = \frac{23}{11}$$

3.(b): Population of village R = 32000(given)

Let the population of village Y be y.

Then, $16:15 = 32000 : y \quad \Rightarrow \quad y = \frac{15 * 32000}{16} = 30000$

4.(b): Population of village Y in 1997 = 30000(given) .

Let the population village V in 1997 be v.

Then, $15:10 = 30000:v \quad \Rightarrow v = \frac{30000 * 10}{15} = 20000.$

Now population of village V in 1998 = 20000 + (10% 0f 20000) = 20000.

$\therefore$Population of village V below poverty line in 1998 = 58% of 22000 = 12760.

5.(a) : Let the total population of all the seven villages in 1997 be N.

Then,population of village R in 1997 = 16% of N = 16/ 100 N

And population of village Z in 1997 = 11% of N =11/100 N

$\therefore$Population of village R in 1999 = {16/100N+(10% of 16/100 N)}=1760/10000 N

and population of village Z in 1999 = {11/100 N-(5% of 11/100 N)} = 1045/10000 N.
Now,population of village R below poverty line for 1999 = 51% of(1760/10000 N)
And population of village Z below poverty line 1999 = 42% of (1045/10000 n)

$$\text{Required ratio} = \frac{51\% \text{ of } (1760/10000\ N)}{42\% \text{ of } (1045/10000\ N)} = \frac{51 * 1760}{42 * 1045} = \frac{2}{1}.$$

39.LINE-GRAPHS

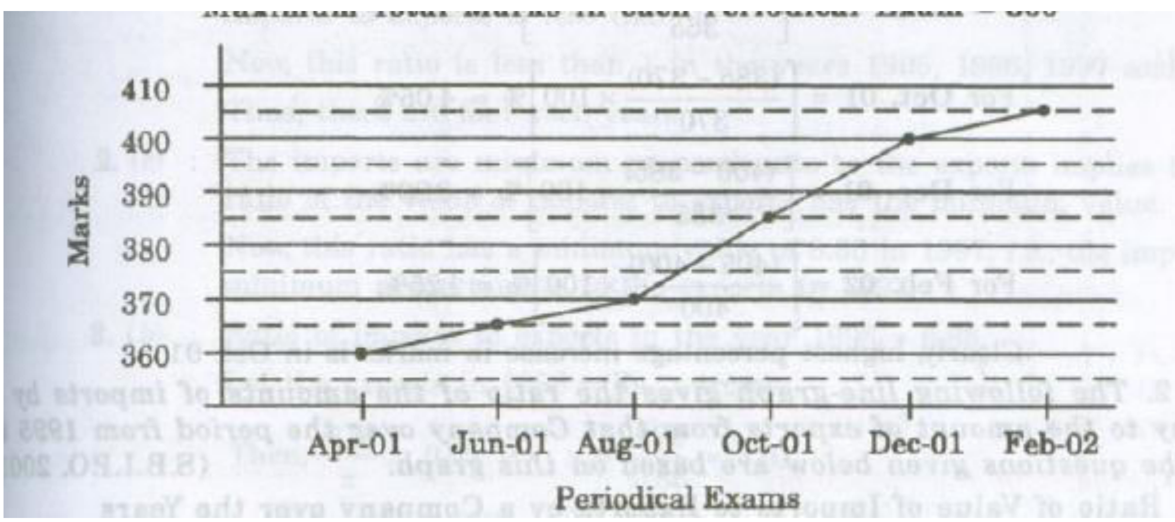
This section comprises of question in which the data collected in a particular discipline are represented by specific points together by straight lines. The points are plotted on a two-dimensional plane taking one parameter on the horizontal axis and the other on the vertical axis. The candidate is required to analyse the given information and thereafter answer the given questions on the basis of the analysis of data.

SOLVED EXAMPLES

Ex. 1. In a school the periodical examination are held every second month. In a session during Apr. 2001 – Mar. 2002, a student of Class IX appeared for each of the periodical exams. The aggregate marks obtained by him in each periodical exam are represented in the line-graph given below. Study the graph and answer the questions based on it.
(S.B.I.P.O 2003)

MARKS OBTAINED BY A STUDENT IN SIX PERIODICAL EXAMS HELD IN EVERY TWO MONTHS DURING THE YEAR IN THE SESSION 2001-02

Maximum Total Marks In each Periodical Exam = 500



1. **The total number of marks obtained in Feb. 02 is what percent of the total marks obtained in Apr. 01?**
   (a) 110% (b) 112.5% (c) 115% (d) 116.5% (e) 117.5%

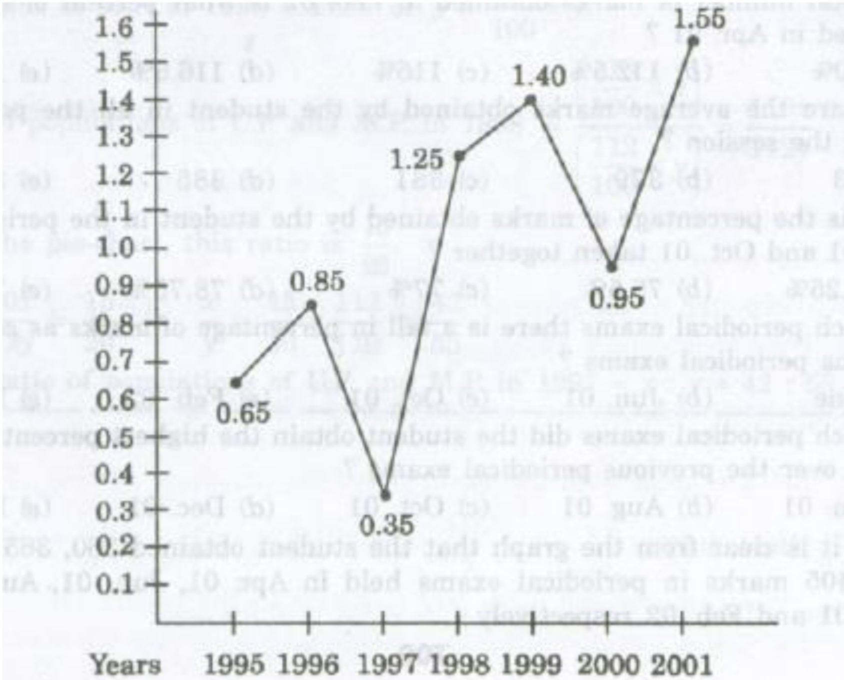
2. **What are the average marks obtained by the student in all the periodical exams of during the session.**
   (a) 373 (b) 379 (c) 381 (d) 385 (e) 389
3. **what is the percentage of marks obtained by the student in the periodical exams of Aug. 01 and Oct. 01 taken together?**
   (a) 73.25% (b) 75.5% (c) 77% (d) 78.75% (e) 79.5%
4. **In which periodical exams there is a fall in percentage of marks as compared to the previous periodical exams?**
   (a) None (b) Jun. 01 (c) Oct. 01 (d) Feb. 01 (e) None of these
5. **In which periodical exams did the student obtain the highest percentage increase in marks over the previous periodical exams?**
   (a) Jun. 01 (b) Aug. 01 (c) Oct. 01 (d) Dec. 01 (e) Feb. 02

Sol. Here it is clear from the graph that the student obtained 360, 365, 370, 385, 400 and 405 marks in periodical exams held in Apr. 01, Jun. 01, Aug. 01, Oct. 01, Dec. 01 and Feb. 02 respectively.

1. (b) : Required percentage = [(405/360)*100] % = 112.5 %
2. (c) : Average marks obtained in all the periodical exams.
   = (1/6)*[360+370+385+400+404] = 380.83 ≈ 381.
3. (d) : Required percentage = [(370+385)/(500+500) * 100] % = [(755/1000)*100]% =75.5%
4. (a) : As is clear from graph, the total marks obtained in periodical exams, go on increasing. Since, the maximum marks for all the periodical exams are same , it implies that the percentage of marks also goes on increasing. Thus, in none of the periodical exams, there is a fall in percentage of marks compared to the previous exam.
5. (c) : Percentage increases in marks in various periodical exams compared to the previous exams are:
   For **Jun. 01** = [(365-360)/360 * 100] % = 1.39 %
   For **Aug. 01** = [(370-365)/365 * 100] % = 1.37 %
   For **Oct. 01** = [(385-370)/370 * 100] % = 4.05%
   For **Dec. 01** = [(400-385)/385 * 100] % = 3.90 %
   For **Feb. 02** = [(405-400)/400 * 100] % = 1.25 %

Ex. 2. The following line- graph the ratio of the amounts of imports by a Company to the amount of exports from that Company over the period from 1995 to 2001. The questions given below are based on this graph. (S.B.I.P.O 2001)

Ratio of value of Import to Export by a Company over the Years



1. **In how many of the given years were the exports more than the imports?**
   a. 1　　b. 2　　c. 3　　d. 4

2.The imports were minimum proportionate to the exports of the Company in the year:
a..1995　b.1996　c.1997　d.2001

3.If the imports of the Company in 1996 was Rs.272 crores, the exports from the Company in 1996 was:
a. Rs.370 crores　b.Rs.320 crores　c.Rs.280 crores
d.Rs.275 crores　e.Rs.264 crores

4.What was the percentage increase in imports from 1997 to 1998?
a. 72　b.56　c.28　d.None of these　e.Data inadequate

5.If the imports in 1998 was Rs.250 crores and the total exports in the years 1998 and 1999 together was Rs.500 crores, then the imports in 1999 was:
a.Rs.250 crores　b.Rs.300 crores　c.Rs 357 crores
d.Rs 420 crores　e.None of these

Sol: 1. d : The exports are more than the imports implies that the ratio of value of imports to exports is less than 1.
Now, this ratio is less than 1 in the years 1995,1996,1997 and 2000.
Thus, there are four such years.

2. c: The imports are minimum proportionate to the exports implies that the ratio of the value of imports to exports has the minimum value.

Now, this ratio has a minimum value of 0.35 in 1997, i.e., the imports are minimum proportionate to the exports in 1997.

3. b: Ratio of imports to exports in the years 1996=0.85.

Let the exports in 1996=Rs.320 crores.

Then,272/x =0.85 implies x=272/.85 = 320.

Exports in 1996 = Rs.320 crores.

4. e: The graph gives only the ratio of imports to exports for different years. To find the percentage increase in imports from 1997 to 1998, we require more details such as the value of imports or exports during these years. Hence, the data is inadequate to answer this question.

5. d: The ratio of imports to exports for the years 1998 and 1999 are 1.25 and 1.40 respectively.

Let the exports in the year 1998 = Rs. x crores

Then, the exports in the year 1999=Rs(500-x) crores.

1.25=250/x implies x=250/1.25=200

Thus the exports in the year 1999=Rs. (500-200)crores=Rs.300 crores

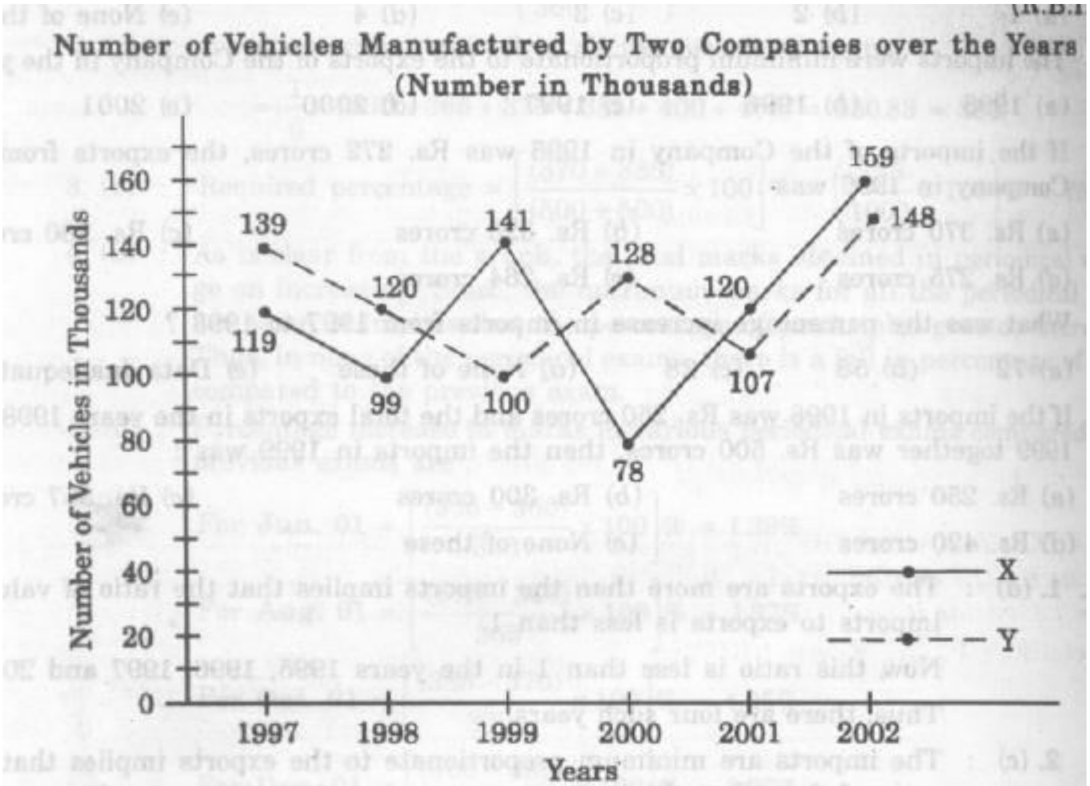
Let the imports in the year 1999=Rs. y crores

Then, 1.4=y/300 implies y=(300*1.4)=420.

Imports in the year 1999=Rs.420 crores.

Ex.3.Study the following line-graph and answer the question based on it.

Number of vehicle Manufactured by Two Companies over the Years
(Numbers in thousands)



1.What is the difference between the total productions of the two Companies in the given years?

a. 19000 b. 22000 c.26000 d.28000 e.29000

2.What is the difference between the numbers of vehicles manufactured by Company Y in 2000 and 2001?

a.50000 b.42000 c.33000 d.21000 e.13000

3.What is the average number of vehicles manufactured by Company X over the given period? (rounded off to the nearest integer)

a.119333 b.113666 c.112778 d.111223 e. None of these

4.In which of the following years, the difference between the productions of Companies X and Y was the maximum among the given years?

a.1997 b.1998 c.1999 d.2000 e.2001

5.The production of Company Y in 2000 was approximately what percent of the production of Company X in the same year?

a.173. b.164 c.132 d.97 e.61

Sol: From the line-graph it is clear that the productions of Company X in the years 1997,1998,1999,2000,2001 and 2002 are 119000,99000,141000,78000,120000and 159000 respectively and those of Company Y are 139000,120000,100000,128000,107000 and 148000 respectively.

1. (c) : Total production of Company X from 1997 to 2002
   = 119000+99000+141000+78000+120000+159000 = 716000
   and total production of Company Y from 1997 to 2002
   =139000+120000+100000+128000+107000+148000=742000
   Difference=742000-716000=26000.

2. (d) : Require difference = 128000-107000 = 21000.
3. (a) : Average number of vehicles manufactured by Company X
   = (91/6)* (119000 + 99000 + 141000 + 78000 + 120000 + 159000) = 119333.

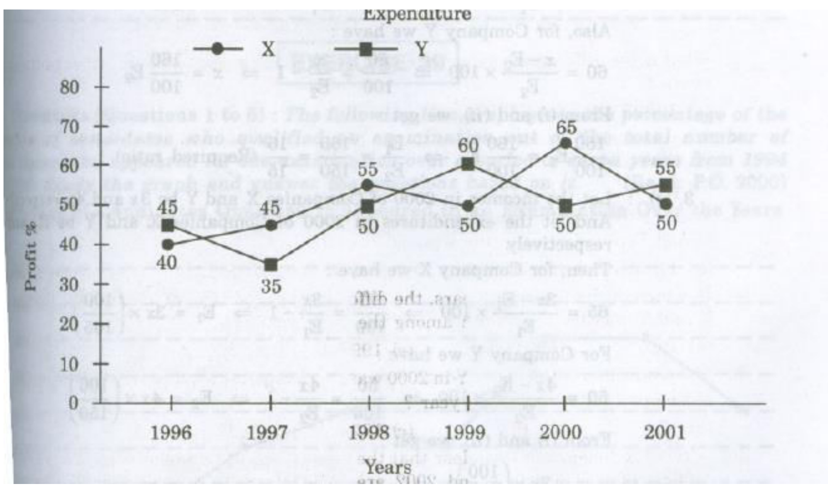
4. (d) : The difference between the production of Companies X and Y in various years are.
   For 1997 = (139000 – 119000) = 20000;
   For 1998 = (120000 – 99000) = 21000;
   For 1999 = (141000 – 100000) = 41000;
   For 2000 = (128000 – 78000) = 50000;
   For 2001 = (120000 – 107000) = 13000;
   For 2003 = (159000 – 148000) = 11000;
   Clearly, maximum difference was in 2000.

5. (b) : Required percentage = [(128000/78000)* 100] % = 164 %.

Ex. 4. The following line-graph gives the percent profit earned by two Companies X and Y during the period 1996 – 2001. Study the line – graph and answer the questions that are based on on it.

Percentage Profit Earned by Two Companies X and Y over the Given years

% profit/ loss = [(Income – Expenditure) / Expenditure] * 100



1. If the expenditure of Company Y in 1997 was Rs. 220 crores, what was its income in 1997?

(a). Rs. 312 crores (b). Rs. 297 crores (c) Rs. 283 crores (d) Rs. 275 crores (e)Rs.261 crores

2.If the incomes of the two companies were equal in 1999,then what was the ratio of expenditure of Company X to that of company Y in 1999 ?

(a) 6:5 (b) 5:6 (c) 11:6 (d) 16:15 (e) 15:16

3.The incomes of the companies X and Y in 2000 were in the ratio of 3:4 respectively.What was the respective ratio of their expenditures in 2000?

(a) 7:22 (b) 14:19 (c) 15:22 (d)27:35 (e) 33:40

4.If the expenditure ofcompanies X and Y in 1996 were equal and the total income of the two companies in 1996 was Rs.342 crores, what was the total profit of the twocompanies together in 1996 ? (Profit = Income – Expenditure)

(a) Rs.240crores (b) Rs.171crores (c) Rs.120crores (d) Rs.102crores (e)None of these.

5.The expenditure ofcompany X in the year 1998 was Rs.200 crores and the income Company X in 1998 was the same as its expenditure in 2001 was:

(a) Rs.465crores (b)Rs.385crores (c)Rs.295crores (d)Rs.255crores

Sol.1.(b) : Profit percent ofcompany Y in 1997=35.

Let the income of company Y in 1997 be Rs.x crores

Then, $35 = \frac{x-220}{220} \times 100 \Rightarrow x = 297$

$\therefore$ Income of company Yin 1997 = Rs.297crores

2.(d): Let the incomes of the twocompanies X and Yin 1999 be Rs.x and let the Expenditures of companies X and Y in 1999 be E_1 and E_2 respectively

Then, for Company X we have:

$$50 = \frac{x-E1}{E1} \times 100 \Rightarrow \frac{50}{100} = \frac{x}{E1} - 1 \Rightarrow x = \frac{150}{100} E1$$

Also, for the Company Y we have:

$$60 = \frac{x-E2}{E2} *100 \Rightarrow \frac{60}{100} = \frac{x}{E2} - 1 \Rightarrow x = \frac{160}{100} E2$$

From (i) and (ii),we get

$$\frac{150}{100} E1 = \frac{160}{100} E2 \Rightarrow \frac{E1}{E2} = \frac{160}{150} = \frac{16}{15} \text{(Required ratio)}$$

3.(c):Let the incomes in 2000 of companies X and Y be 3x and 4x respectively.And let the expenditure in 2000 of companies X and Y be E1 and E2 respectively.

Then, for company X we have:

$$65 = \frac{3x-E1}{E1} *100 \Rightarrow \frac{65}{100} = \frac{3x}{E1} - 1 \Rightarrow E1 = 3x * \left(\frac{100}{165}\right)$$

For company Y we have:

$$50 = \frac{4x-E2}{E2} *100 \Rightarrow \frac{50}{100} = \frac{4x}{E2} - 1 \Rightarrow E2 = 4x * \left(\frac{100}{150}\right)$$

From (i)and(ii) we get:

$$\frac{E1}{E2} = \frac{3x*(100/165)}{4x*(100/150)} = \frac{3*150}{4*165} = \frac{15}{22} \text{(Required ratio)}$$

4.(d):Let the expenditures of each of the Companies X and Y in 1996 be Rs.xcrores.And let the income of Company X in 1996 be Rs.zcrores so that the income of Company Y in 1996 =Rs.(342-z)crores.

Then,for company X we have:

$$40 = \frac{z-x}{x} *100 \Rightarrow \frac{40}{100} = \frac{z}{x} - 1 \Rightarrow x = \frac{100z}{140}$$

Also for company Y we have:

$$45 = \frac{(342-z)-x}{x} *100 \Rightarrow \frac{45}{100} = \frac{(342-z)}{x} - 1 \Rightarrow x = \frac{(342-z)*100}{145}$$

From(i)and (ii) we get:

$$\frac{100z}{140} = \frac{(342-z)*100}{145} \Rightarrow z = 168$$

Substituting z=168 in (i),we get: x=120

∴ Total expenditure of companies X and Y in 1996=2x=Rs.240crores.

Total income of companies X and Y in 1996=Rs.342 crores.

∴Total profit =Rs.(342-240)crores =Rs.102 crores

5.(a): Let the income of company X in 1998 be Rs.x crores.

Then, $55 = \frac{x-200}{200} * 100 \Rightarrow x = 310.$

$\therefore$ Expenditure of Company X in 2001= Income of company X in 1998 = Rs.310crores

Let the income of company X in 2001 be Rs.z crores

Then, $50 = \frac{z-310}{310} * 100 \Rightarrow z = 465.$

$\therefore$ Income of company X in 2001 = Rs.465 crores.